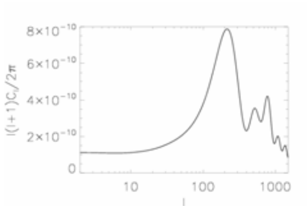
Microwave Background Interferometry on the Spherical Sky

E.F. Bunn (U. Richmond) and M. White (U.C. Berkeley)

Introduction

Interferometers have been extremely important in past comic microwave background (CMB) observations. For a variety of reasons, particularly the ability to control systematic errors, interferometry is a promising technique for upcoming measurements in both temperature and polarization.

Constraining theories with the CMB often involves measuring the angular power spectrum with sharp resolution in Fourier (multipole) space, requiring observations over a large fraction of the sky with subdegree angular resolution. That is, we require resolution $\Delta\ell \sim 1$ with $\ell \gg 1$.



In interferometers, these goals are likely to be met by mosaicking many pointings, each with a small enough field of view. The flat-sky approximation is appropriate for individual pointings but not for the mosaic. A full spherical harmonic analysis is possible in this situation, but it is computationally much better to use a formalism that takes advantage of the flat-sky approximation where appropriate.

References

Hobson et al., MNRAS, 275, 863 (1995)
Hobson & Magueijo, MNRAS, 283, 1133 (1996)
Myers et al., ApJ, 591, 575 (2003)
White et al., ApJ, 514, 12 (1999).

Results shown here are based on **astro-ph/0606454, ApJ, 655, 21 (2007).**

This work was supported by NSF grant 0507395.

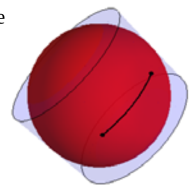
Cylindrical Sky Approximation

A key ingredient in comparing interferometric CMB observations with theoretical models is the visibility covariance matrix for a given theory, with elements

$$M_{jk} = \langle V_j^* V_k \rangle,$$

where the visibilities V_j, V_k may correspond to different pointings and different baselines.

We can compute each matrix element by approximating the celestial sphere as a cylinder containing the two pointing centers. This allows the use of Fourier transforms instead of spherical harmonic expansions. As long as the individual pointings have small beam sizes, this is an excellent approximation for arbitrary pointing separations.



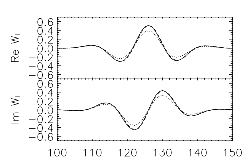
Plane Wave Approximation

It is possible to approximate the exact spherical harmonic expansion by approximating each spherical harmonic as a plane wave. Near the equator,

$$Y_{\ell m} \approx N_{\ell m} e^{im\phi} \begin{Bmatrix} \cos(n_{\ell m} z) \\ \sin(n_{\ell m} z) \end{Bmatrix}.$$

with $m^2 + n_{\ell m}^2 = \ell(\ell+1)$.

However, this approximation is never better and is in some cases worse than the cylindrical Fourier method. The reason is that the spherical harmonics are unevenly distributed over the (m,n) plane, so that in some cases the visibility covariance matrix is dominated by a small number of spherical harmonics.

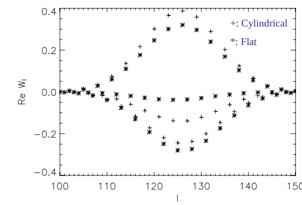


Window function for a single baseline covariance.

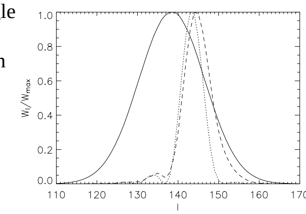
Solid: Exact
Dashed: Cylindrical
Dotted: Plane-wave

Results

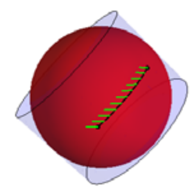
Window function for the covariance between two identical baselines with $\vec{u} = 20\hat{\phi}$, separated by 120°. The Gaussian beam width is 5° (FWHM=12°).



Window functions for a single pointing (solid), and for a 10x10 mosaic of pointings in the flat-sky (dotted) and cylindrical (dashed) approximations. The cylindrical approximation is indistinguishable from the exact calculation (not shown). All baselines have u=22 and point in the azimuthal direction. Pointings are in a square grid centered on the equator with separation 5°.



The main difference between the flat and cylindrical window functions is *baseline rotation*. Correlations between visibilities with very different pointing centers are strong only when the baselines the baselines are nearly identical. To judge whether two baselines are the same, parallel transport one baseline to the other along the great circle joining the two.



The green baselines shown in this picture are parallel in this sense, but baselines with identical components in spherical coordinates would not be.

Conclusions

We have developed a method for calculating the visibility covariance matrix for interferometric CMB observations with mosaics covering arbitrarily large fractions of the sky. The method is quite accurate, as long as the field of view of the individual pointings is relatively small (FWHM < 20°). The method extends naturally to polarization observations.