Optimal Phase Shift Sequences for a Bolometric Interferometer for Microwave Background Polarimetry

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Background

Cosmic microwave background (CMB) polarization gives us a method of measuring the imprint of gravitational waves during the inflationary epoch. Thus, measuring the polarization of the CMB is an important test of inflation, and should also give us information about the mass distribution of the universe at the unprecedented age of 10^{-30} seconds.

Considerable effort is underway to develop technologies that will allow detection of the very weak CMB polarization signal. One approach, currently being developed by the MBI collaboration (Brown, Wisconsin, Richmond) is bolometric interferometry. The approach potentially combines sensitive detectors, excellent control of systematic errors, and scalability to the ~10^3 detector systems that will ultimately be required.

Phase Shifting

With incoherent detectors such as bolometers, the signal in an interferometer is the sum of the E-fields of the antenna array. Thus, for a 3-antenna array, the received signal will be

$$S = (E_1 + E_2 + E_3)^2$$
$$= \sum_i E_i^2 + 2(E_1E_2 + E_1E_3 + E_2E_3)$$

To determine the interferometric visibilities, we need a way of isolating the individual cross terms E_iE_j. This is done by introducing variable phase shifts at each input, which introduces a time-varying signal. Signals at different times can then be combined to cancel out all terms other than the desired cross term. Our research deals with determining the minimum possible time steps needed to make a list of signals capable of solving for every cross term in a given array, thus maximizing the efficiency of the satellite. We consider both 180º and 90º phase shifts.

Solving for Cross Terms

Let *n* be the number of input horns. The state of the phase shifters at any time is an *n*-dimensional vector with values ±1 (for 180º shifts) or ±1 ±*i* (for 90º shifts). If a set of m time steps is applied, the sequence can be represented as an *n* x *m* matrix. To be able to extract all *n*(*n*-1)/2 cross terms with minimum noise, we need the matrix to satisfy the condition that all products of pairs of rows are orthogonal. To operate the instrument with maximum efficiency, we wish to minimize the number of time steps *m* for any given number *n* of inputs, or equivalently, we wish to find the maximum value of *n* for any given *m*.

MBI prototype

We assume that m is a power of 2 (for 180º shifts) or 4 (for 90º shifts). While this will not necessarily lead to the optimal solution, we conjecture that the solutions given by our method at least closely approximate the optimal solution. Mathematically, results from the properties of Hadamard matrices, used in the calculation, show that the solution (at least in the 180 degree case) must be a multiple of 2, and all attempts to find solutions not given as a power of 2 at low numbers of time steps were unable to do so.

Since the number of possible states of the array increases dramatically with the number of time steps, much of the research done focused on means of improving the searching algorithms. Our improvements centered on two areas: (1) Techniques to eliminate essentially isomorphic matrices efficiently from our search, and (2) developing a technique to focus on the most efficient strains. The latter allowed us to calculate the likely optimal phase shifts to determine the cross terms in arrays of up to 45 antennas for 180 degree phase shifting, and 60 antennas for 90 degree phase shifts.

Results

Number of time steps	Maximum antennas
2	2
4	3
8	4
16	6
32	7
64	9
128	12
256	16
512	21

Number of time steps	Maximum antennas
4	2
16	4
64	8
256	15
1024	24
4096	40
16384	66

180 degree case, left.
90 degree case, above.

These tables show the maximum number of antennas for various values of *m*. A typical ground-based or balloon-borne experiment would be likely to have *n* ~ 10, while arrays with *n* = 64 are under consideration for a possible satellite-borne instrument. In the later case, large numbers of time steps are required.

Future research

Currently, we have only considered arrays in which each pair of antennas gives an independent visibility. If the array has some symmetry, however, some pairs will give equivalent visibility. The pattern of phase shifts can then be chosen to read out equivalent visibilities simultaneously, reducing both the noise in those visibilities and the required number of time steps. Our future research will center on solutions that take advantage of these redundancies for arbitrary geometries of antennas.

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