Proof of MVT in small words

Introduction

Our goal is to write the proof of Mean Value Theorem in monosyllabic words. Dr. Evelyn Lamb is a mathematician and blogger. In her blog *Roots of Unity* on the *Scientific American* website, she wrote Rolle's Theorem in monosyllabic words. Our challenge is to write Mean Value Theorem in the same way. We first stated the theorem and proof in regular words, then wrote our small words version.

Below is the link of Dr. Evelyn Lamb's blog, check it out if you want to see the small words version Rolle's Theorem.[1]

In Regular Words, for Context

Mean Value Theorem:

If $f : [a, b] \to \mathbb{R}$ is continuous on $[a, b]$ and differentiable on (a, b), then there exists a point $c \in$ (a, b) where $f'(c) = \frac{f(b)-f(a)}{b-a}$.

Proof in regular words:

The line through $(a, f(a))$ and $(b, f(b))$ is $y = \frac{f(b)-f(a)}{b-a}(x - a) + f(a)$. The difference d between this line and the function itself is $d(x) = f(x) - \left[\left(\frac{f(a)-f(b)}{b-a}\right)(x - a) + f(a)\right]$ for $x \in [a, b]$. Since f and y are continuous, d is continuous on $[a, b]$ by Theorem 4.3.4 (iii). Distance d is differentiable on (a, b) by Theorem 5.2.4 (i). Note that $d(a) = 0 = d(b)$. By Rolle's Theorem, there is some point $c \in (a, b)$ such that $d'(c) = 0$. Since $d'(x) = f'(x) - \frac{f(b)-f(a)}{b-a}$, it follows that $0 = f'(c) - \frac{f(b)-f(a)}{b-a}$. So at the point c provided by Rolle's Theorem, $f'(c) = \frac{f(b)-f(a)}{b-a}$.

In Small Words

What the Math Fact says:

Let f be a map from a closed length of reals (the length from a to b that has both a and b in it) to the reals. Let K be the set of points more than a and less than b. If the slope of the graph of f can be found at all points in K, and if f does not jump on K, then there is a point c in K where the slope of f at c is the same as the slope of the line through the points $(a, f(a))$ and $(b, f(b))$

Proof in small words:

Let y be a line through $(a, f(a))$ and $(b, f(b))$. For each x that is more than a and less than b, d is the gap from $f(x)$ to $y(x)$.

[1] https://blogs.scientificamerican.com/roots-of-unity/a-proof-of-the-math-fact-of-rolle-in-short-words/

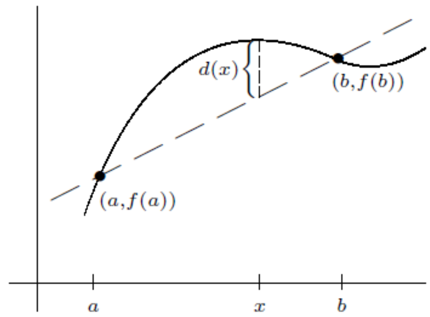

Figure 1: From book, page 157

Since the slopes of $f(x)$ and y can be found at all points in K, by Math Fact 5.2.4 (i), the slope of $d(x)$ can be found at all points in K.

Since $f(x)$ does not jump on K and y does not jump as well, we can use Math Fact 4.3.4 (iii), which says that $d(x)$ too does not jump. Note that $d(a)$ is the same as $d(b)$, and is the same as zilch, since f and y both go through $(a, f(a))$ and $(b, f(b))$. This means we can now use Rolle's Math Fact, which says there is a c in K where the slope of d is zilch.

Since the slope of d at x is the same as the when we take the slope of y at x from the slope of f at x, then the slope of d at point c will be the slope of y at c from the slope of f at c. Since the slope of d at c is zilch, then the slope of f at c is the same as the slope of y.