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Band pattern of commensurate modulated periodic structures

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Abstract: Analogies with physical phenomena indicate that modulation of a material or geometrical parameter of a periodic structure enriches its original band structure. The present work aims to provide an insight into the band-splitting phenomenon in the case of commensurate modulation for a parallel-plate waveguide technology-based geometry. A modulated one-dimensional parallel-plate waveguide signal integrity structure is numerically analysed to exhibit the appearance of band splitting and new bandgaps. The modulation mechanism has a potential in dispersion engineering, as it allows controlling the number and position of the electromagnetic bandgaps and the in-band characteristics of the field propagation. Generation of modes with negative group velocities for a given frequency band is also achievable by this technique.

1 Introduction

Periodic structures have recently found many applications in problems concerning applied electromagnetics. Two-dimensional (2D) repetition of a unit cell built in microstrip or parallel-plate waveguide structure technology has proven useful for filtering, antenna feeding and design in view of obtaining prescribed radiation patterns, improving signal integrity in high-speed digital and mixed signal printed circuit boards and integrated circuits, cloaking and so on [1–4].

Wave propagation in periodic structures is described by the dispersion diagram (DD), which can feature (or not) electromagnetic bandgaps (EBGs) [5, 6]. Certain analogies to physical phenomena show that EBGs split when the modulation of a certain material or geometrical parameter is superposed to the initial periodicity of the structure [7]. The band splitting shows a regular pattern when the period of modulation is commensurate with the initial period of the structure. The pattern becomes a fractal structure for the incommensurate case. Group-theoretical deductions applied to the EBGs of a 2D or 3D structure contain useful information but do not give a complete solution, so detailed numerical solutions of the eigenvalue equations are necessary.

The present work aims to provide an insight into the band-splitting phenomenon in the case of commensurate modulation for a parallel-plate waveguide technology-based geometry. The modulation of microstrip or parallel-plate waveguide structure devices has potential applications to dispersion engineering issues as, e.g. design of electronically switched surfaces, devices with flatter group velocity, control of negative phase and group velocities and so on. In Section 2, we discuss commensurate modulation of periodic structures and show that, while impact of modulation on the unit cells is mathematically tractable, the determination of the exact band structure must rely on numerical simulation (or experiment). In Section 3, we demonstrate the application of the proposed discussion on a 1D structure that is periodic in two orthogonal directions, built in strip-line technology, which has been initially devised for signal integrity applications. This structure has been selected for illustration because it has been optimised for having a large EBG. Results reported in Section 3 confirm the behaviour of modulated structures as presented in Section 2. Conclusions are drawn in Section 4.

2 Commensurate modulation of 1D periodic structures

Consider an infinite 1D periodic structure composed of a unit cell of length d that extends along the x-axis. The electromagnetic waves propagating through this structure at a specified frequency ω are Bloch waves ψ. A Bloch wave has the special property that waves at one end of a unit cell are related to waves at the other unit end by

$$\psi(x+d) = \mathrm{e}^{-\mathrm{j}kd}\psi(x) \tag{1}$$

For a given frequency f, the transmission characteristics of the structure depend on the nature of Bloch wave's wavenumber $k = \beta - \mathrm{j}\alpha$ in (1). When k has a non-zero imaginary part, the wave is evanescent. In what follows we are interested in propagating waves for which the imaginary part is zero. Plotting the (k, f) relation for real k yields the DD of the structure [5].

The DD of a periodic structure is determined by solving an eigenequation of the form:

$$\boldsymbol{AV} = \lambda \boldsymbol{V} \tag{2}$$

where $\boldsymbol{A}$ is a transmission matrix depending on the structure of the unit cell, λ is the eigenvalue and $\boldsymbol{V}$ is the eigenvector. For example, in the case of infinite periodic structures, $\boldsymbol{A}$ can be the $\boldsymbol{ABCD}$ matrix of the unit cell [6, 8]. By setting $\lambda = \mathrm{e}^{-\mathrm{j}kd}$, (2) can be solved for different frequencies. For some frequencies, the waves are evanescent or a solution to the eigenproblem (2) does not exist. These frequencies belong to EBGs and so there are no propagating waves at these frequencies. To find the propagating modes, we are searching for distinct solutions of (2) for $-\pi \le kd \le \pi$. The propagating solutions are periodic with respect to the normalised wavenumber kd, with period 2π, and can be grouped into several continuous functions of k called modes [5, 6, 9]. EBGs might occur between some modes.

We now consider that one geometrical or material parameter of the periodic medium is subjected to a second periodically impressed motif, of period d', which may be commensurable or not with the initial period d. In the commensurate case, we have

$$pd' = qd = \delta \tag{3}$$

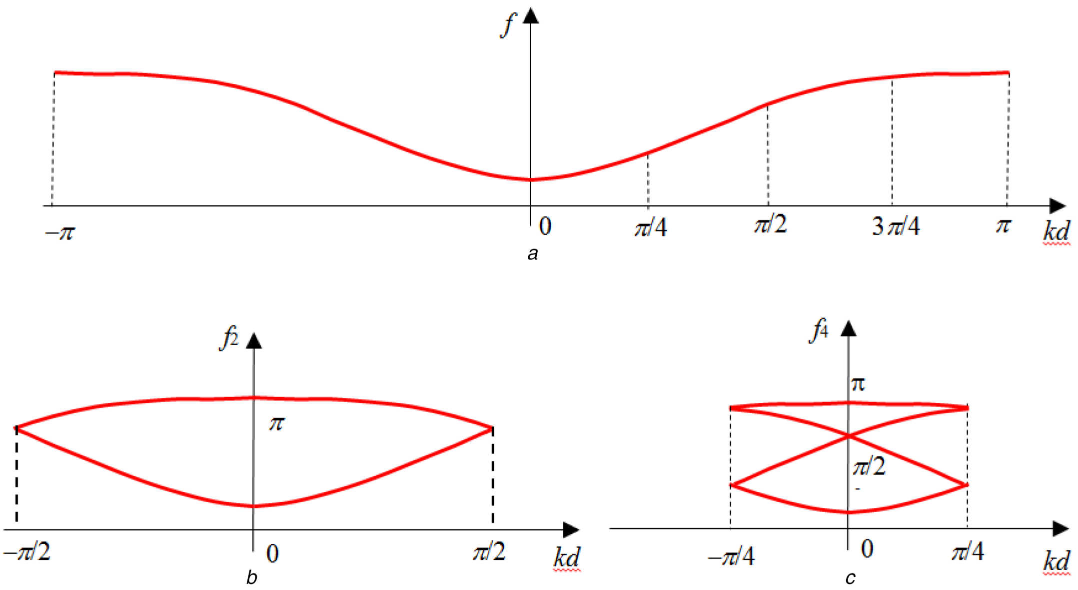

Fig. 1 *One mode of the unmodulated structure (frequency versus normalised wavenumber, one period) calculated with*
(a) (2) (see text), ***(b)*** (5) for $q = 2$, ***(c)*** (5) for $q = 4$. The values of π and $\pi/2$ along the vertical axis in *(b)* and *(c)* correspond to the phase difference kd in *(a)*

where p and q are two coprime integers. The whole modulated structure consists of the repetition of the unit cell with the large period $\delta = qd$.

This periodic alteration of the initial structure can be referred to as *modulation*. For example, the length ℓ of a certain element in the large period unit cell can have a sinusoidal variation

$$\ell_n = \ell + \ell_m \sin\left(\frac{2\pi}{d'}x + \phi\right)\Big|_{x = n(p/q)d'} \tag{4}$$

Similar variations can be imposed to material parameters or to lumped elements entering into the structure of the large unit cell. The relation between the competing periods $d, (d')$ and δ plays an important role in the response of the structure to an incoming electromagnetic wave. Previous evidence shows that the modulation has a profound impact on the unmodulated EBGs [10, 11]. One of the effects is the splitting in a (large) number of new EBGs [7].

The analysis of the EBGs for the modulated structure is drawn from the study of 1D polyatomic molecules [5, 10]. We do not intend to convey the idea that the properties of a 1D model are straightforwardly generalisable for explaining 2D or 3D structures. To obtain the specific positions of the splitting of different bands for a 1D structure we used numerical simulations. However, the 1D analysis is useful to emphasise the basic band splitting phenomenon introduced by the modulation.

Let us consider first the unmodulated structure, of period d, as consisting of a larger period $\delta = qd$, for which the transmission matrix is $\boldsymbol{A}^q$ and the eigenequation reads

$$\boldsymbol{A}^q \boldsymbol{V} = \mu \boldsymbol{V} \tag{5}$$

The eigenvalues resulting from (2) and (5) are related by $\mu = \lambda^q$. Since in the case of real wavenumbers, we have the trivial equality $[\mathrm{e}^{-\mathrm{j}(kd + (2n\pi/q))}]^q = \mathrm{e}^{-\mathrm{j}qkd}$, distinct solutions for modes are obtained only on the interval $-(\pi/qd) \le k \le (\pi/qd)$. We denote by $f(k)$ a mode provided by (2), where f stands for the frequency and $-(\pi/d) \le k \le (\pi/d)$.

Then f_q, which represents the same mode but provided by (5), is a multivalued function that maps k from $-(\pi/qd) \le k \le (\pi/qd)$ into the set

$$f_q(k) = \left\{ f(k), f\left(k + \frac{2\pi}{qd}\right), \ldots, f\left(k + \frac{2\pi(q-1)}{qd}\right) \right\} \tag{6}$$

Note that the set (6) does not change if we make the substitution $k \to k + (2\pi/qd)$, due to the periodicity of f. Therefore, $2\pi/qd$ is the period of f_q.

The above discussion can be illustrated graphically as shown in Fig. 1. If the mode $f(k)$ has the shape of Fig. 1*a*, then this mode will be represented as shown in Fig. 1*b* for $q = 2$ and as shown in Fig. 1*c* if $q = 4$. Note that the normalised wavenumber kd is marked on the horizontal axis. The branches of the graphs of Figs. 1*b* and *c* are constructed from the graph of Fig. 1*a* by applying (6).

As a second step, let us consider now that a p/q geometrical modulation is applied to the structure as in (4). The new period of the structure is now δ from (3). If the modulation index (ℓ_m/ℓ in (4)) is not very high, then the shape of the modes of the modulated structure will not differ much from the curves introduced above (e.g. in Figs. 1*b* and *c*). However, akin to a similar phenomenon from solid state physics [12], the vanishing derivatives at $k\delta = \pm\pi$ enforce the splitting of modes as shown in Figs. 2*a* and *b*. Every unmodulated mode, called *parent mode*, is broken into q branches called *child modes*. If the parent mode is monotonous and occupies alone a certain frequency range, then a maximum of $[q/2]$ EBGs is introduced by the modulation. The child modes have alternatively group velocities of opposite signs.

The shapes and number of the modes, i.e. of the DD, depend on the modulation parameters p and q. For example, the shapes for $p/q = 1/4$ will differ from those for $p/q = 3/4$.

An example of a structure built with transmission lines and with a known $\boldsymbol{A}$ matrix, [8, p. 547], illustrates the effects of modulation and the appearance of EBGs. The unit cell of Fig. 3*a* consists of a capacitance connected between two segments of lossless transmission line with $d = 3.5$ cm, $C_0 = 1$ pF, characteristic impedance $Z_0 = 50\,\Omega$ and phase velocity on the lossless lines $v_{\mathrm{ph}} = 2.75 \times 10^8$ m/s [6, 8]. The restricted representations for the first four propagating f-modes and f_2-modes are represented in Fig. 3*b*. There are no new EBGs for the f_2-modes, according to the previous general discussion.

However, new EBGs appear if the capacitances of the nth unit cell are modulated

$$C_n = C_0 + C_m \sin\left(\frac{2\pi}{d'}x + \phi\right)\Big|_{x = n(p/q)d'} \tag{7}$$

Fig. 3*c* represents the case $p/q = 1/2$, $C_m = 0.5$ pF and $\phi = 0$.

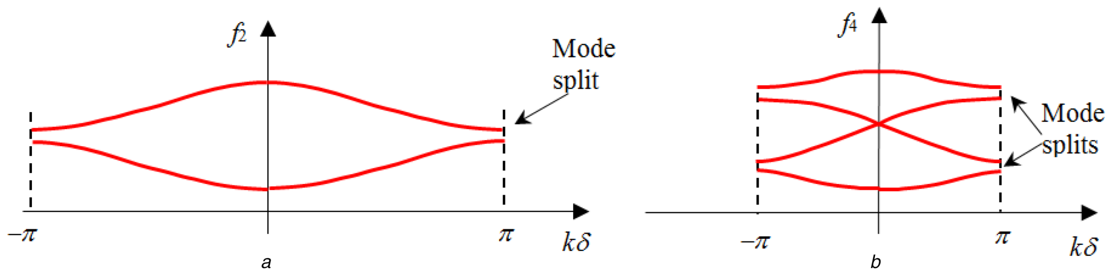

Fig. 2 *Child modes of the modulated structure generated by the parent mode in Fig. 1a (frequency versus normalised wavenumber, one period)* ***(a)*** $q = 2$, ***(b)*** $q = 4$

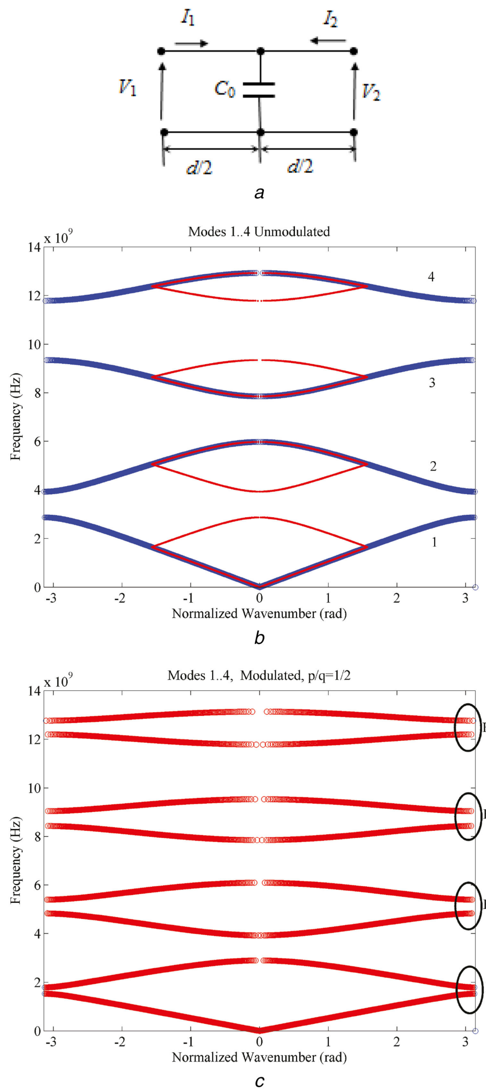

Fig. 3 ***(a)*** *Unit cell,* ***(b)*** *First four propagating f-modes, in blue, are labelled by n = 1, 2, 3 and 4. For a common comparison, all figures are plotted versus the normalised wavenumber kd. Modes f_2, in red, for the $\boldsymbol{A}^2$ matrix covers the normalised interval $[-\pi/2, \pi/2]$ because before modulation both $\boldsymbol{A}$ and $\boldsymbol{A}^2$ have the same period, namely d. There are no new EBGs for the f_2-modes,* ***(c)*** *Similar to Fig. 2a, EBG at π are introduced by modulation with p/q = 1/2*

3 Modulation in parallel-plate waveguide structure technology

The above analysis shows that, by introducing a parameter modulation over q unit cells of an infinite 1D lattice (i) each mode splits and (ii) a maximum of $[q/2]$ additional EBGs occur out of each mode (iii) the fundamental period of the modulated DD is reduced by a factor of q. Can these guiding rules offer insight into DD modification for more complex structures? To this end, we modulated a 1D structure, which consists of a sandwich of two dielectric layers confined between two metal planes. Circular metal patches are inserted periodically at the interface of the dielectrics and are connected to the lower metal plane (ground) by four vias with metal walls. Coin-shaped metal inclusions are inserted into the upper dielectric layer and are connected by cylinders to the upper metal plane. There are four cylinders per unit cell; their centres are at the half of the distance between the patch and top boundary ($z = t_2/2$) Fig. 4 [13]. This structure has been introduced in [14] for signal integrity applications and it has been chosen because it was optimised to provide a large EBG. The same structure is used here as a reference because it allows for an easier demonstration of the modulation band splitting phenomenon. The geometrical and material parameters are reported in Fig. 4. The coins are equally spaced all over the structure and thus form a periodic pattern with period one fourth of that of the structure in the x-direction (with respect to the reference frame in Fig. 4). Repeating the cell on an infinite 2D plane produces the DD from Fig. 5 where only positive wavenumbers are displayed for symmetry reasons. The DD was calculated with a commercial eigenmode solver [15], that solved (2) by imposing periodic boundary conditions in the x- and y-directions and electric boundary conditions on the z-direction. Only waves propagating in the x-direction have been considered for the DD in order to be consistent with the 1D setting of Section 2.

A modulation of $p/q = 1/2$ in (4) was applied to coins radii by keeping an average value $\ell = r_c = 1.8\,\text{mm}$, a modulation amplitude $\ell_m = 0.6\,\text{mm}$ and $\phi = 0$. To illustrate the relation between the DDs obtained before and after modulation, we have first calculated the eigenvalues for the unmodulated structure setting the spatial period at 20 mm, which is twice the distance $d =$ 10 mm of the unit cell in the x-direction. The 'unit cell' is depicted in the inset of Fig. 4. The modes obtained by solving (5) with $q = 2$ by means of the same eigenmode solver are represented in Fig. 5 and bear two indices, the first one indicating the parent mode and the second one its branch resulted by using (5) instead of (2).

After modulation, the marked modes of Fig. 5 will transform into the DD of the modulated structure shown as an inset in Fig. 6. The DD for the modulated coins is shown in Fig. 6. The modes in Figs. 5 and 6 have been marked according to the results provided by Microwave Studio, Computer Simulation Technology, v. 2015 [15]. It can be seen that the eight modes in Fig. 6 are children of the four parent modes of Fig. 5. Note that the horizontal axes have been normalised to d in Fig. 5 and to $\delta = 2d$ in Fig. 6.

When the modulation is imposed, the DD needs to fulfil the zero-derivative condition and so the restricted unmodulated (parent) mode representation must change and show a bent of the curves at the edges of the k-zone. This is visible in Fig. 6, especially in between modes (1, 1) and (1, 2). The adjacent curves thus break at the edges where normalised wavenumbers equal to $\pm\pi$ and new EBGs occur. New EBGs do not occur between higher-order modes because they intersect in between each other. However, the mode splitting is visible. Note also that mode (1, 2) in Fig. 6 has a negative group velocity. This mode is related to mode 1 of the original structure from Fig. 4, which exhibits positive group velocity over all its extensions. The negative group velocity is a consequence of the mirroring of the parent mode with respect to the vertical axis.

The initial structure in Fig. 5 has an EBG between modes 1 and 2 in the frequency range of (4.26-6.49) GHz. The modulated

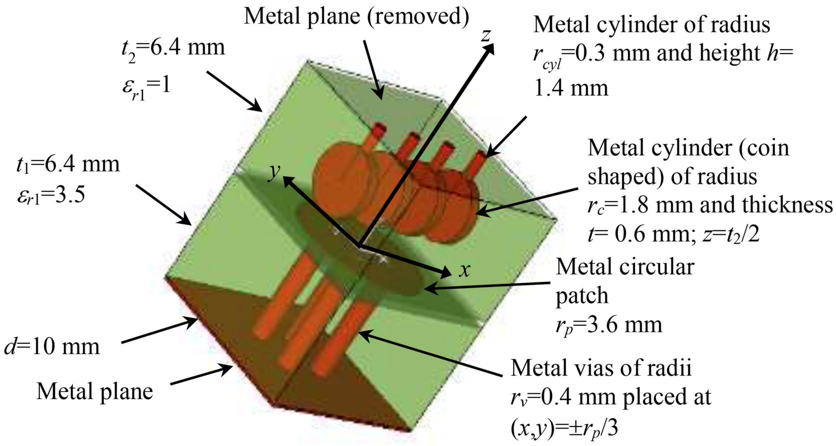

Fig. 4 *CAD model of the basic unit cell*

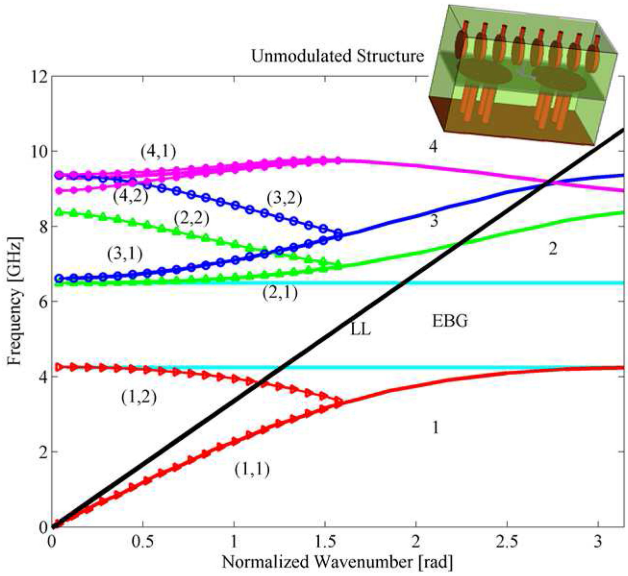

Fig. 5 *Frequencies versus normalised wavenumber calculated by the eigenmode solver for the unmodulated structure. Continuous lines correspond to the unit cell in Fig. 4, q = 1, whereas the marked curves correspond to q = 2 which is the unit cell from the inset*

structure has two EBGs, one in the interval (4.26-6.50) GHz, that almost coincides with the former one, and a newly introduced one (EBG1 in Fig. 6) in the interval (3.06-3.52) GHz. The appearance of the EBG1 between the child modes of the former mode 1 is a direct effect introduced by the modulation. In conclusion, the relationship between modulated and unmodulated cases is presented for structures that can be described by field equations. As a future work, geometry modulation will be applied to electronically switched surfaces in order to tailor the EBG structure of the states [16, 17].

4 Conclusions

We have discussed the phenomenon of band-splitting that occurs in periodic structures with a superimposed modulation that is commensurate with the initial period of the structure. We have shown that, if the ratio of the modulation and initial periods is p/q, then every mode that is present in the DD of the initial structure gives rise to q modes in the DD of the modulated one. Furthermore, if the initial mode is monotonous, then so are the newly introduced ones, but monotonicity alternates from one mode to the next one and so does the group velocity. New EBGs appear between these new modes.

The modulation of periodic structures has potential applications in dispersion engineering due to the control it provides on band-structure and group velocity. Analysis of specific applications is under consideration and will be the subject of future works.

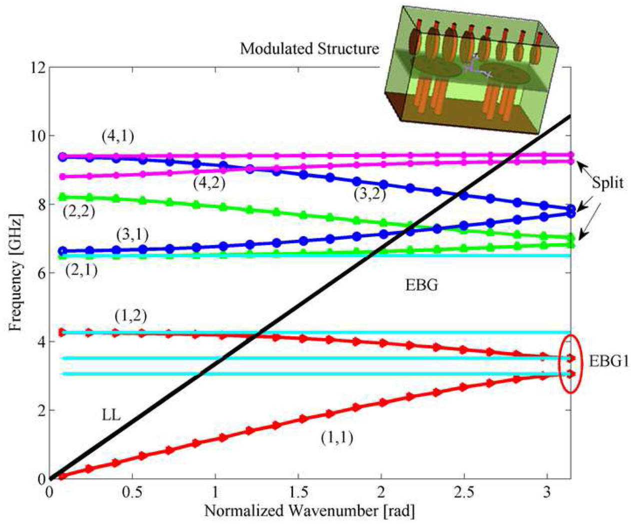

Fig. 6 *DD for the modulated unit cell shown in the inset. Modulation is done on the q = 2 unit cell of Fig. 5*

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