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Exploiting Analytic Transfer Matrices to Engineer Resonances in Patterned Dielectric Periodic Surfaces

Aldo De Sabata[(1)], Ovidiu-Zeno Lipan [(2)], and Ladislau Matekovits[(1,3)]
(1) Politehnica University Timişoara, Timişoara, Romania
(2) Department of Physics, University of Richmond, Richmond, VA, USA
(3) Politecnico di Torino, Torino, Italy

***Abstract*—Designing periodic structures with prescribed resonant frequencies is a difficult task if a theoretical model is unavailable. In this paper, it is shown that analytic transfer matrices can be used to engineer resonance properties of an all-dielectric periodic 2D structure that exhibits a sharp transition in reflection-transmission properties at resonance frequencies in the range 1500-1800 GHz, dependent on the angle of incidence of electromagnetic waves between 5 and 85 degrees. The operation of the structure is assessed by full-wave simulation. The proposed all-dielectric periodic surface has potential application to sensors.**

I. Introduction

All-dielectric 2D periodic structures have gained attention of the research community in the last years due to applications to spatial filtering, reflectarrays, polarization control, sensors etc., for frequencies ranging from GHz to THz [1-4].

One of the main issues in designing periodic structures consists of the control of resonant frequencies as a function of the geometrical and material parameters of the unit cell. An analytical solution, apt to bring some contribution to this issue, has been proposed recently. The solution is based on a closed-form formula for the transfer matrix relating the coefficients of the Bloch-Floquet expansions of the fields on either side of a periodic, inhomogeneous dielectric slab [5, 6].

In this paper, an all-dielectric silicon slab with periodically drilled holes is shown to have a remarkably sensitive response to the angle of incidence of electromagnetic waves in TE polarization. This behavior is highly dependent on the variation of the resonance frequency of the structure with the incidence angle, which has been assessed in this work by means of both transfer matrix calculation and by full-wave simulation with a specialized software [7]. The high sensitivity of the response opens the way for potential applications to sensors.

The structure and the simulation results are presented in Section II, and the comparison with results derived analytically from the calculated transfer matrix is considered in Section III. Conclusions are drawn in the last Section.

II. Presentation of the Structure

The dielectric, patterned structure has been obtained from a bulk slab of silicon (ε_r=11.9), of 2 μm thickness, with square shaped drilled holes, having an edge of 85.82 μm, and with a periodicity of 120 μm in two orthogonal directions. The unit cell is represented in Fig. 1. The frequency range of interest for this application is 1500-1800 GHz.

When a TE polarized electromagnetic wave is incident on the surface, at an angle θ, propagating in the *-z* direction with respect to the reference frame in Fig. 1, a sudden change in the transmission-reflection properties occurs due to a Fano resonance whose parameters are dependent on θ. At the resonance frequency, the structure exhibits a sharp reflection peak, while, at neighboring frequencies it is almost transparent. Furthermore, the Bloch mode TE(-1,0) is launched at the resonant frequency.

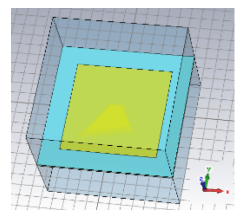


Figure 1. Geometry of the unit cell and reference frame.

These properties have been revealed by the simulations that have been performed in TE polarization, for angles of incidence from 5 to 85 degrees, with a step of 5 degrees. Forty Bloch-Floquet modes have been considered at both the input port (Z_{max}) and output port (Z_{min}). The amplitudes of evanescent modes have been sampled at a distance of 1 μm from the surface of the slab.

Some results are reported in Fig. 2, for angles of incidence θ between 20 and 50 degrees. For the remaining values of θ, the results are similar.

The reflection coefficient is displayed in Fig. 2 (a), the transmission coefficient in Fig. 2 (b), and the coupling coefficient of mode TE(-1,0) (denoted as 9 by the simulation program) is represented in Fig. 2 (c). The coupling coefficients of the remaining evanescent modes are below -10 dB.

The results reported in Fig. 2 confirm the existence of a high sensitivity of the resonant frequency with respect to the angle of incidence and of an abrupt change in reflection-transmission properties of the structure at those frequencies.

III. Assessment through Transfer Matrix

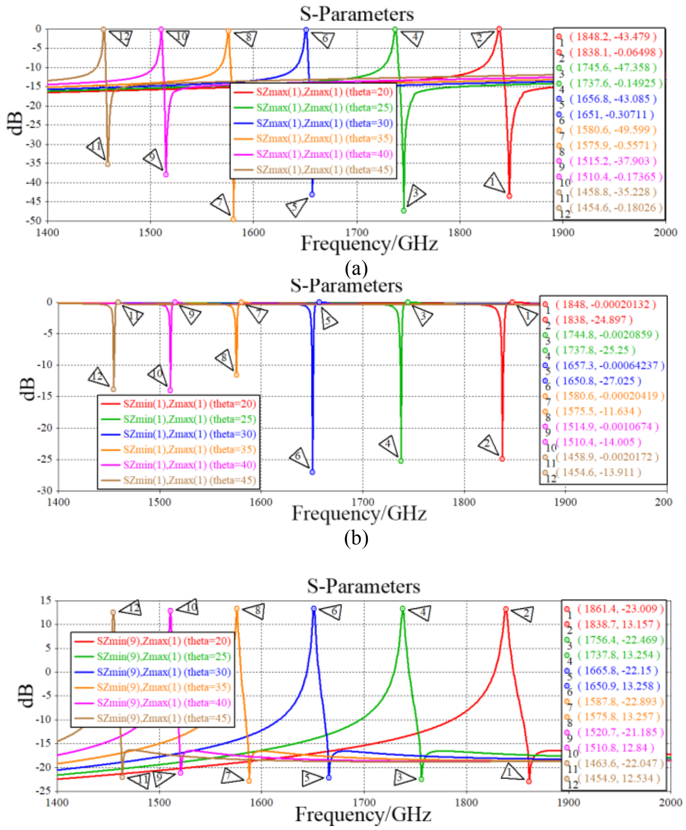
The procedure used for calculating the transfer matrix is presented in [5, 6]. As already mentioned, the transfer matrix **T** relates the coefficients of the Bloch-Floquet expansion of the field at Z_{min} to those at Z_{max}. The form of the transfer matrix is

$$\mathbf{T} = \begin{bmatrix} \mathbf{T}^{++} & \mathbf{T}^{+-} \\ \mathbf{T}^{-+} & \mathbf{T}^{--} \end{bmatrix}. \qquad (1)$$

In (1), "+" indicates propagation in the $+z$ direction (with respect to the reference frame in Fig. 1), while "-" indicates propagation in the $-z$ direction. The transmission scattering matrix is readily obtained as

$$\mathbf{S}_{\text{min,max}} = \left(\mathbf{T}^{--}\right)^{-1} \ . \qquad (2)$$

Thus, the zeros of $S_{Zmin(1)Zmax(1)}$ are the roots of the cofactor of the first element from $\mathbf{T}^{--}$ [5].



(c)`

Figure 2. Simulation results: (a) reflection coefficient; (b) transmission coefficient; (c) coupling coefficient for mode TE(-1,0).

To calculate the resonance frequencies, the first five TE modes (ordered by Rayleigh frequencies) have been used, namely (0,0), (±1,0), and (0,±1). The theoretical results are presented in Fig. 3, together with the resonance frequencies provided by the CST. The agreement between the results obtained by two different methods validates both their correctness, and the fact that the transfer matrix can be used for conveniently designing similar structures appropriate to operate in other frequency bands.

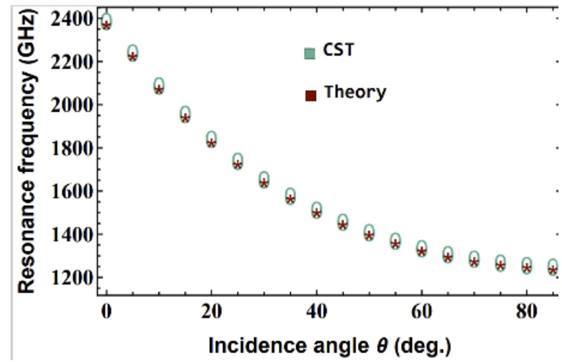


Figure 3. Resonance frequencies calculated through the transfer matrix (Theory) and by simulation (CST).

IV. Conclusions

An all-dielectric 2D periodic structure, consisting of a slab of silicon with periodically drilled square-shaped holes has been considered in this paper. It has been shown that a series of Fano resonances are present in the interval 1500-2800 GHz that are sensitive to the angle of incidence of TE polarized electromagnetic waves between 5 and 85 degrees. While the structure is almost transparent in the given frequency range, a sharp reflection and the launch of a confined Bloch mode occur when the angle of incidence and the frequency of the wave match a resonance of the periodic structure. This property opens the way for applications to sensors.

The resonance frequencies have been found by full-wave simulation and cross-checked by analytic calculation based on a closed form formula for the Pendry-MacKinnon transfer matrix. The results of both methods are in agreement, supporting their accuracy. Therefore, optimization methods can be devised for designing structures with prescribed specifications by means of an analytic approach, which is known to give better insight and provide reduced computation time as compared to brute-force simulations.

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