CORRELATION FUNCTION

The mean of the product of the random force $X(t)$ at times t' and t'', namely

$$\langle X(t')\, X(t'')\rangle$$

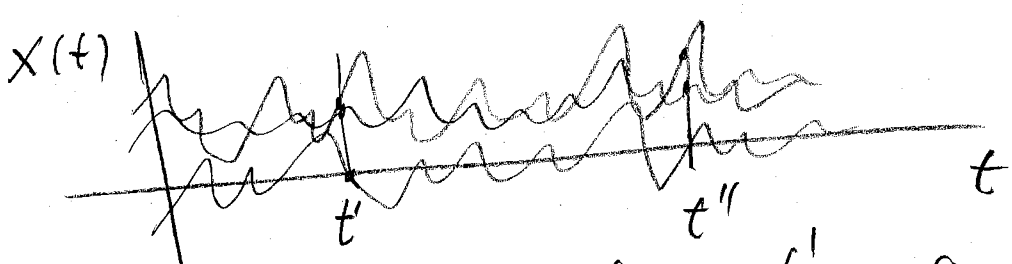
is called TWO-TIME CORRELATION FUNCTION

A multitime correlation function is a correlation function of the form

$$\langle X(t')\, X(t'')\, X(t''') \cdots \rangle .$$

The moast important correlation function is the two-time correlation function, because, as we will see, the multitime correlation function can be computed out of the two-time correlation function for an important class of stochastic processes.

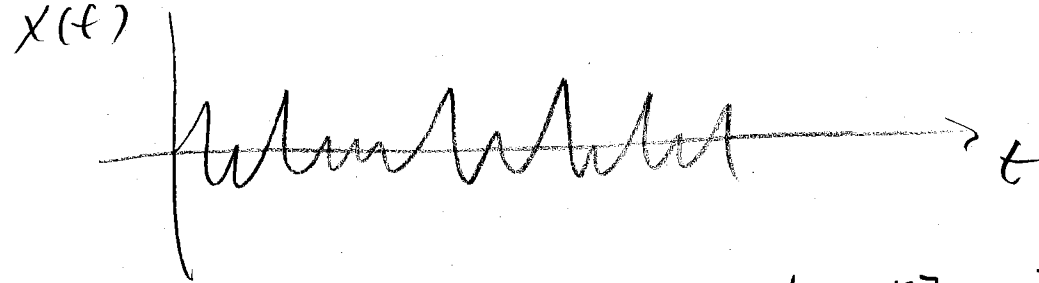
Two-time correlation function



If t'' is far away from t', a natural hypothesis is that the noise at $X(t')$ is independent of the noise at $X(t'')$ so

$$\langle X(t')\, X(t'')\rangle = \langle X(t')\rangle \langle X(t'')\rangle$$

Another natural hypothesis for a noise is that the mean is zero



$$\langle X(t')\rangle = 0 \quad \text{at any } t'$$

So

$$\langle X(t')\, X(t'')\rangle = \langle X(t')\rangle \langle X(t'')\rangle = 0$$

if $t'' > t'$.

If t'' is close to t' we can say that the noise has some memory and thus

$$\langle X(t')\, X(t'')\rangle \neq 0 \quad \text{for } t'' \text{ close to } t'$$

Also, the two-point correlation function will depend only on the relative position of t' and t'', that is only on $t''-t'$

$$\langle X(t')\, X(t'')\rangle = \text{a function of } t''-t'$$

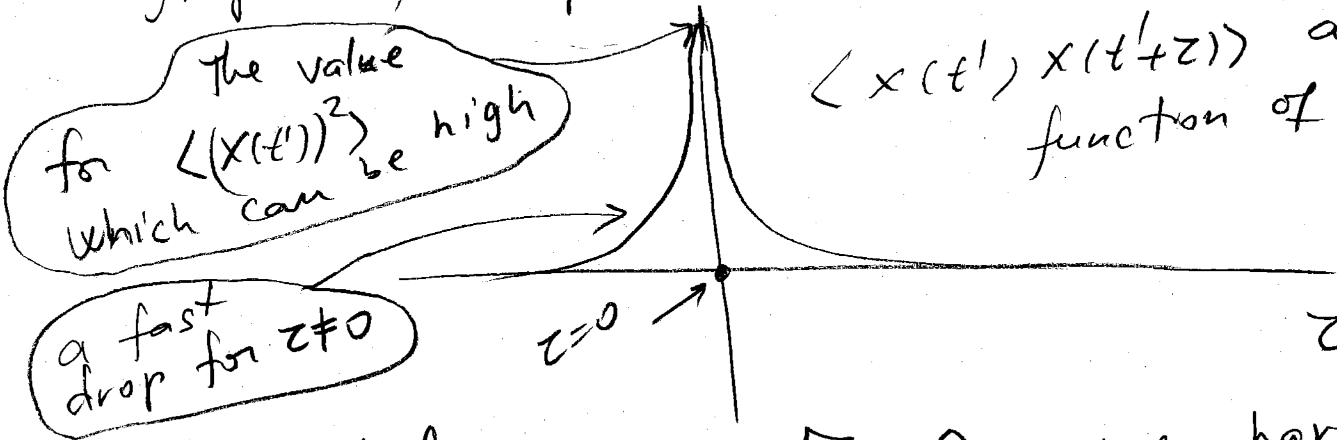
If we denote thus $t''-t' = \tau$ we obtain a function of one variable, τ,

$$\langle X(t')\, X(t'+\tau)\rangle$$

keeping t' fixed.

All the above hypothesis can be graphically represented as

$\langle x(t')\, x(t'+\tau)\rangle$ as a function of τ.



Note that, when $\tau = 0$, we have

$$\langle x(t')\, x(t'+\tau)\rangle = \langle (x(t'))^2\rangle$$

which is <u>NOT ZERO</u> and also is

<u>NOT SMALL</u>.

Remember that for the Brownian motion, the speed (which was our model of noise) is such that

$$\langle (\text{Brownian speed})^2\rangle = \frac{2D}{dt}$$

and if $dt \to 0$ we get

$$\langle (\text{Bownian speed})^2\rangle \to \infty$$

So, for noise, we can have

$$\langle (X(t'))^2\rangle \longrightarrow \infty .$$

For the Langevin equation

$$\dot{v} = -\gamma v + \underbrace{X(t)}_{\text{random}}$$

the meaning of "SOLVE IT" is:

Find $\langle v(t) \rangle$

$\langle v^2(t) \rangle$

$\vdots$

$\langle v^m(t) \rangle$.

If $X(t)$ is <u>NOT</u> RANDOM, than the meaning of "SOLVE IT" is:

Find $\boxed{v(t)}$

and you are done.

When $X(t)$ <u>is</u> RANDOM we say that we need to solve (that is find $\langle v \rangle, \langle v^2 \rangle, \ldots$) a STOCHASTIC DIFFERENTIAL EQUATION.

Now we start to find $\langle v^2 \rangle$.

$$\langle v^2 \rangle = \left\langle \left(v(t_0)\, e^{-\gamma(t-t_0)} + \int_{t_0}^{t} e^{-\gamma(t-t')} \frac{X(t')}{m} dt' \right)^2 \right\rangle$$

$$= \left[v(t_0)\, e^{-\gamma(t-t_0)} \right]^2 + \left\langle v(t_0) e^{-\gamma(t-t_0)} \int_{t_0}^{t} e^{-\gamma(t-t')} \frac{X(t')}{m} dt' \right\rangle$$

$$+ \left\langle \int_{t_0}^{t} e^{-\gamma(t-t')} \frac{X(t')}{m} dt' \int_{t_0}^{t} e^{-\gamma(t-t'')} \frac{X(t'')}{m} dt'' \right\rangle$$

We need

$$\langle X(t') \rangle = ?$$

$$\langle X(t')\, X(t'') \rangle = ?$$

Now for

$\langle v^3 \rangle$

we need $\langle X(t')\, X(t'')\, X(t''') \rangle = ?$

Must be included in the definition of NOISE

$$\boxed{\text{To move forward we need to study } \langle X(t') \rangle,\ \langle X(t')\, X(t'') \rangle \text{ etc.}}$$

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