Lecture

Bose - Einstein condensation

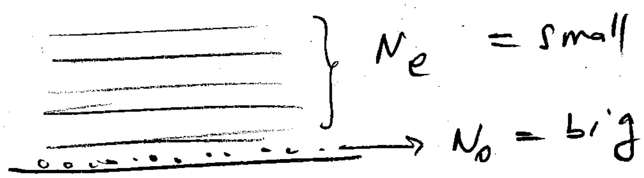
When $\rho \lambda^3$ is not small we cannot use the above approximation. We need to go back to

$$\frac{N-N_0}{V} = \frac{1}{\lambda^3} g_{\frac{3}{2}}(z) \qquad \text{with } 0 < z \leq 1$$

$$\frac{P}{kT} = \frac{1}{\lambda^3} g_{\frac{5}{2}}(z)$$

and study the situation for which

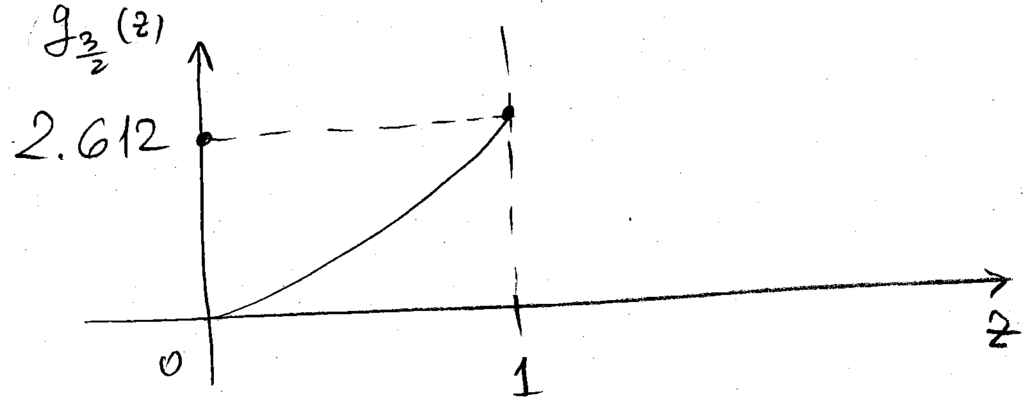
$$N_0 \approx N$$



$$N = N_e + N_0$$

↑ because N_0 is big, almost all particles are in the ground state

The function $g_{\frac{3}{2}}(z)$ has a peculiar behavior, it increases monotonically with z and is <u>BOUNDED</u>



The maximum value of $g_{\frac{3}{2}}(z)$ is at $z=1$

$$g_{\frac{3}{2}}(z) = 1 + \frac{1}{2^{\frac{3}{2}}} + \frac{1}{3^{\frac{3}{2}}} + \dots \equiv \zeta\left(\frac{3}{2}\right) \simeq 2,612$$

↑ Riemann zeta function

because

$$\frac{N - N_0}{V} = \frac{1}{\lambda^3} g_{\frac{3}{2}}(z)$$

we conclude that

$$\frac{N - N_0}{V} \leq \frac{1}{\lambda^3} g_{\frac{3}{2}}(1) = \frac{2,612}{\lambda^3}$$

N_e, the number of particles in the excited states

Important conclusion

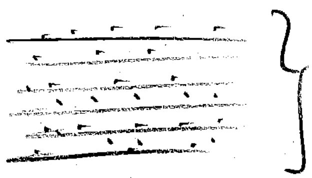
For a given volume V and temperature T, the number of particles in the excited states is **bounded**

$$N_e \leq V \frac{(2\pi m k T)^{\frac{3}{2}}}{h^3} \zeta\left(\frac{3}{2}\right)$$

What happens if the total number N of particles in the system exceeds the above limiting value

$$N > V \frac{(2\pi m k T)^{\frac{3}{2}}}{h^3} \zeta\left(\frac{3}{2}\right)$$

The excited states will recive the maximum allowed value $V \frac{(2\pi m k T)^{\frac{3}{2}}}{h^3} \zeta\left(\frac{3}{2}\right)$ while the rest will have to occupy the ground stat, which can accomodate an unlimited number of particle !



Excited states at full capacity $V \frac{(2\pi m k T)^{\frac{3}{2}}}{h^3} \zeta\left(\frac{3}{2}\right)$

Ground state can collect an unlimited number of particles.

The condition for Bose-Einstein condensation is thus

$$N > V T^{\frac{3}{2}} \frac{(2\pi m k)^{\frac{3}{2}}}{h^3} \zeta\left(\tfrac{3}{2}\right)$$

↑ Hold V, T constant and increase N

or

$$T < T_{critic} = \frac{h^2}{2\pi m k}\left(\frac{N}{V \zeta\left(\frac{3}{2}\right)}\right)^{\frac{2}{3}}$$

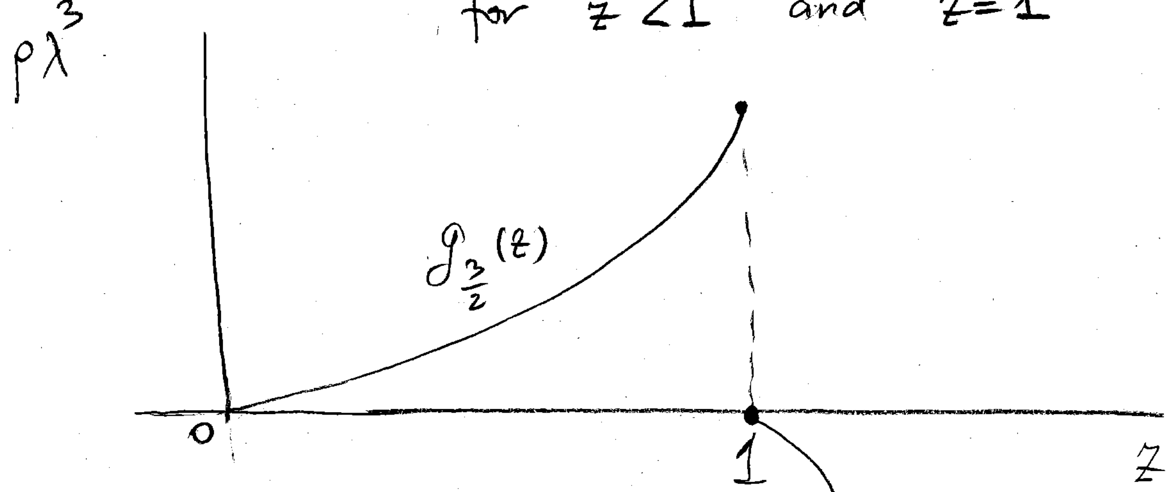
↑ Hold N and V constant and vary T

At $T < T_c$ the system will be in a mixture of two phases

(1) a <u>normal phase</u>, consisting of N_e particles distributed over the excited states ($\epsilon \neq 0$)

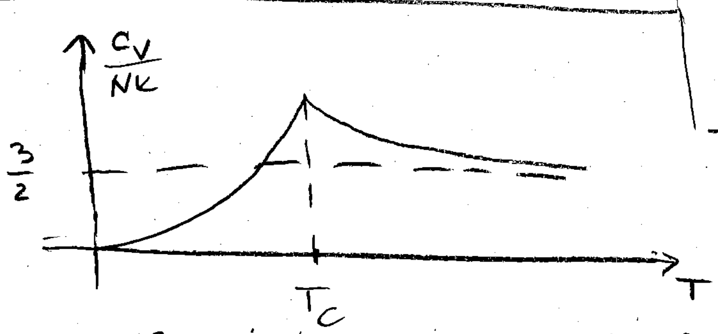
(2) a <u>condensed</u> phase, consisting of N_0 particles accumulated in the ground state ($\epsilon = 0$)

Comparison of BOSE GAS thermodynamics for $z < 1$ and $z = 1$



$z < 1$ ($T > T_c$)	$z = 1$ ($T < T_c$) DEGENERATE BOSE GAS, BOSE-EINSTEIN CONDENSATION
$\frac{N_e}{V} = \frac{1}{\lambda^3} g_{\frac{3}{2}}(z)$	$\frac{N_e}{V} = \frac{1}{\lambda^3} g_{\frac{3}{2}}(1)$
$\frac{P}{kT} = \frac{1}{\lambda^3} g_{\frac{5}{2}}(z)$	$\frac{P}{kT} = \frac{1}{\lambda^3} g_{\frac{5}{2}}(1)$
$\frac{U}{N} = \frac{3}{2} \frac{kT}{\rho\lambda^3} g_{\frac{5}{2}}(z)$	$\frac{U}{N} = \frac{3}{2} \frac{kT}{\rho\lambda^3} g_{\frac{5}{2}}(1)$
$\frac{C_v}{Nk} = \frac{15}{4} \frac{1}{\rho\lambda^3} g_{\frac{5}{2}}(z) - \frac{9}{4} \frac{g_{\frac{3}{2}}(z)}{4 g_{\frac{1}{2}}(z)}$	$\frac{C_v}{Nk} = \frac{15}{4} \frac{1}{\rho\lambda^3} g_{\frac{5}{2}}(1)$

here $g_{\frac{5}{2}}(1) = \zeta\left(\frac{5}{2}\right) = 1.342$



SPECIFIC HEAT OF THE PERFECT BOSE GAS

The Nobel Prize in Physics 2001

"for the achievement of Bose-Einstein condensation in dilute gases of alkali atoms, and for early fundamental studies of the properties of the condensates"

Eric A. Cornell	**Wolfgang Ketterle**	**Carl E. Wieman**
1/3 of the prize	1/3 of the prize	1/3 of the prize
USA	Federal Republic of Germany	USA
University of Colorado, JILA Boulder, CO, USA	Massachusetts Institute of Technology (MIT) Cambridge, MA, USA	University of Colorado, JILA Boulder, CO, USA
b. 1961	b. 1957	b. 1951

FIG. 9. MIT faculty in ultralow-temperature atomic physics. Dan Kleppner, W.K., Tom Greytak, and Dave Pritchard look at the latest sodium BEC apparatus [Color].

- September 29 1995

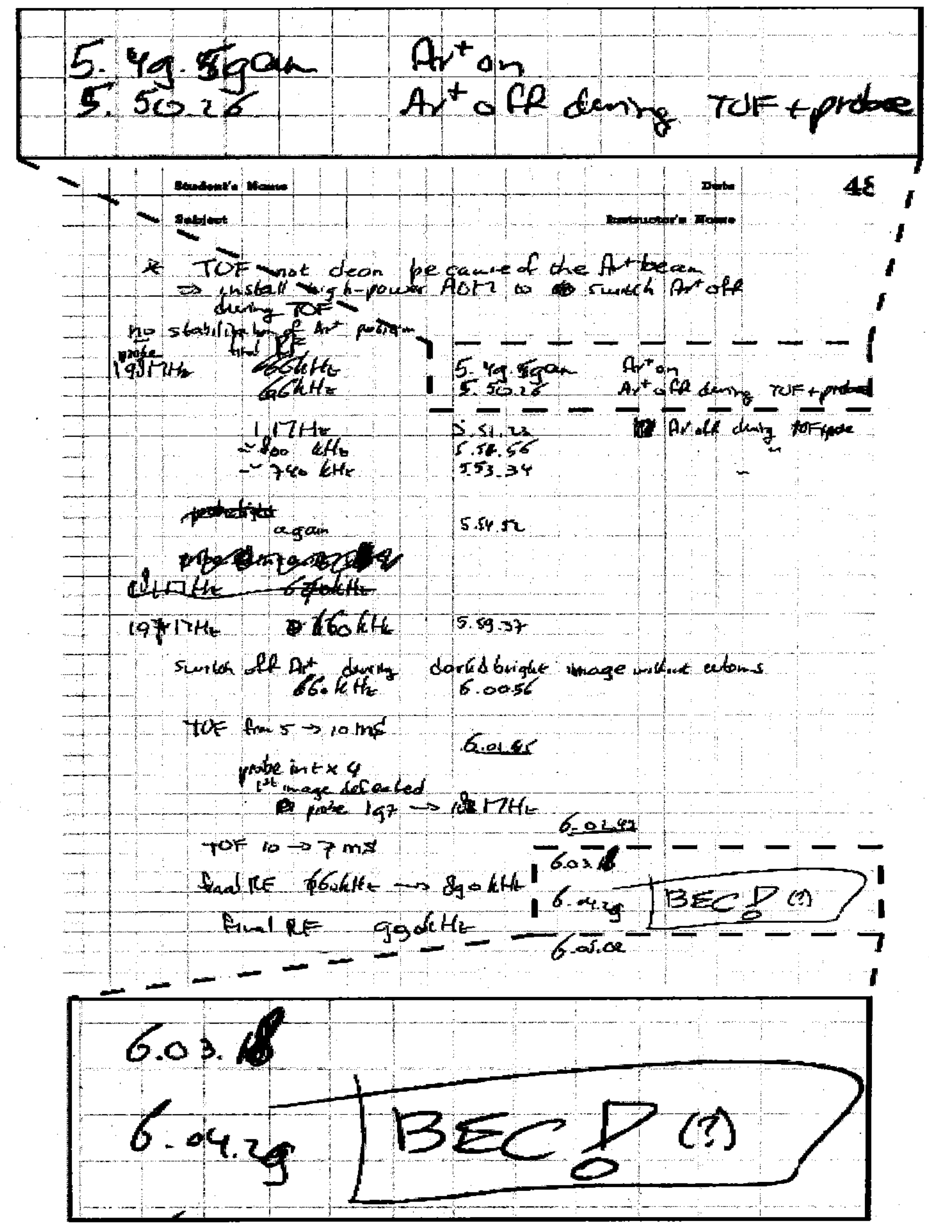


FIG. 6. One page of the lab book during the night of September 29, 1995, when BEC was first observed at MIT. The handwriting is by Klaasjan van Druten. At 5:50 a.m., we had installed a new acousto-optical modulator to switch off the optical plug (Ar ion laser beam). Fifteen minutes later, we had the first definitive evidence for BEC in sodium.

FIG. 11. Comparison of a laser cooling and BEC experiment. The first photograph shows the author in 1993 working on the Dark SPOT trap. In the following years, this laser cooling experiment was upgraded to a BEC experiment. The second photograph shows the same apparatus in 2001 after many additional components have been added [Color].

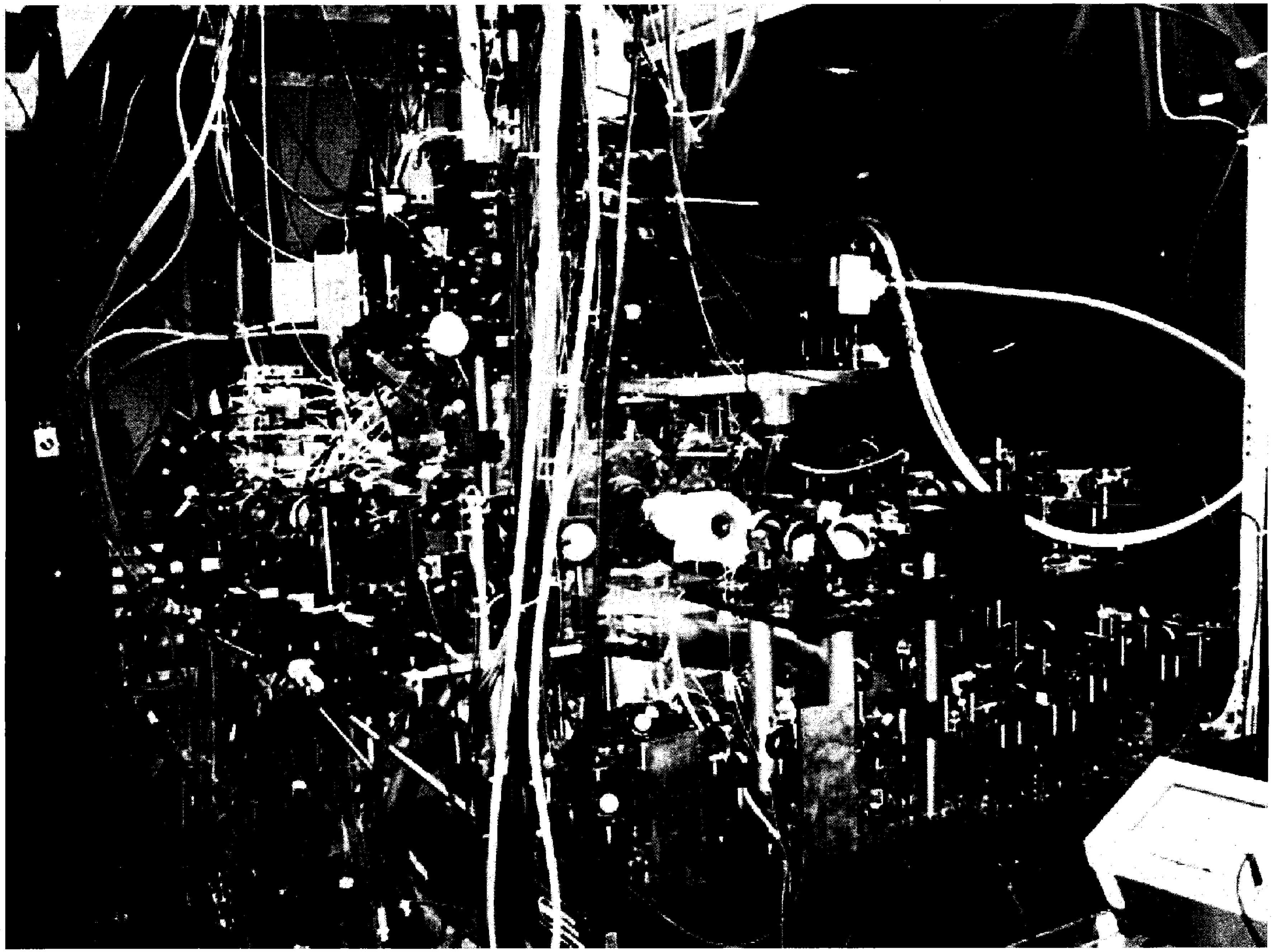


FIG. 11. (*Continued.*)

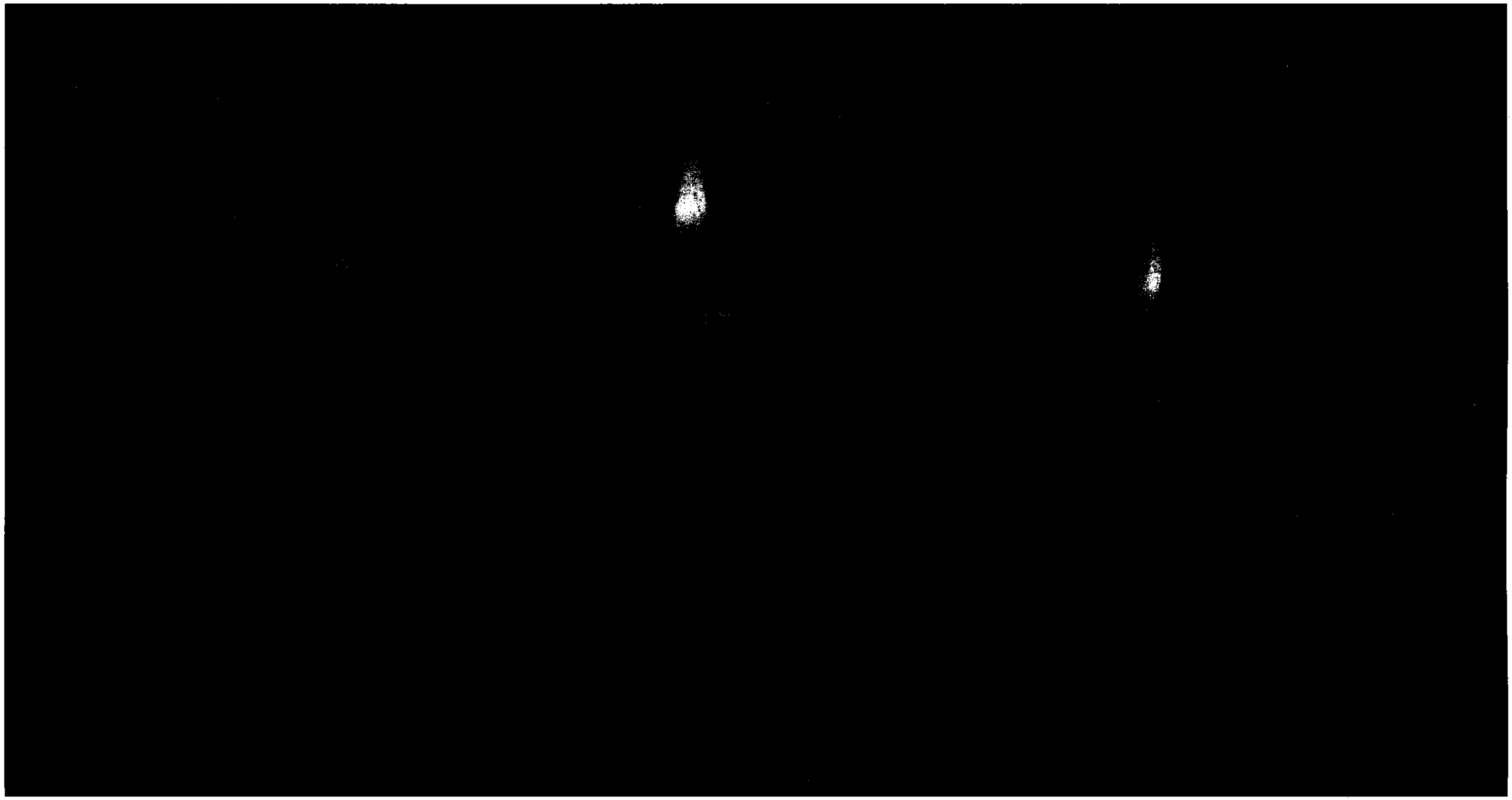


FIG. 7. Observation of Bose-Einstein condensation by absorption imaging. Shown is absorption vs two spatial dimensions. The Bose-Einstein condensate is characterized by its slow expansion observed after 6 ms time of flight. The left picture shows an expanding cloud cooled to just above the transition point; middle: just after the condensate appeared; right: after further evaporative cooling has left an almost pure condensate. The total number of atoms at the phase transition is about 7×10^5, the temperature at the transition point is 2 μK [Color].

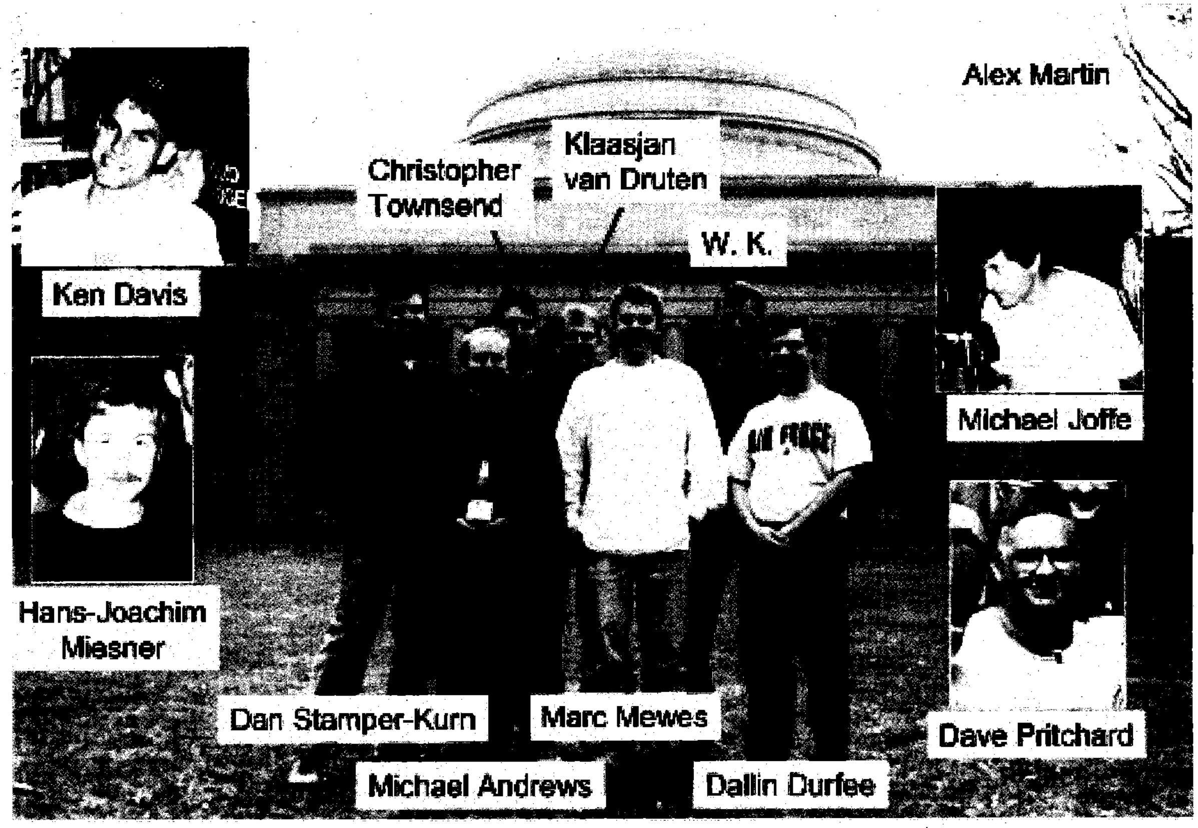


FIG. 16. Team photo. This photo was taken in early 1996 in front of the MIT dome. The bottle of champagne was emptied to celebrate BEC in the cloverleaf trap. Names and photos of other collaborators during the period 1992–1996 have been added.