

Lecture

Practical use of statistical physics computations: ADIABATIC DEMAGNETIZATION.

Goal: find a procedure to lower the temperature
We obtained the entropy $S(T, N, B)$ as a function of the thermodynamic state (T, N, B) :

$$S(T, N, B) = Nk \ln \left[2 \cosh \left(\frac{\mu B}{kT} \right) \right] - \frac{N\mu B}{T} \tanh \left(\frac{\mu B}{kT} \right)$$

Also we found that the

$$\text{Heat Exchange} = T ds$$

When the paramagnetic salt is isolated from the heat bath there will be no exchange of heat so

$$ds = 0$$

We conclude that the entropy is constant

$S = \text{CONSTANT}$
if the system is isolated from the heat bath. Remember that ds was the thermodynamic change in entropy when the state changed.

From the entropy formula we see that the entropy depends on T and B only through their ratio

$$S(T, N, B) = Nk f\left(\frac{B}{T}\right)$$

with f a function that is not very ugly. Actually with the notation

$$x = \frac{\mu B}{kT}$$

the entropy can be written as

$$S = Nk \left[\ln 2 + \ln(\cosh x) - x \tanh(x) \right]$$

a form that reveals the function f .

Now with

$$\mu \approx \mu_{\text{BOHR}} = \frac{e\hbar}{2m_e} = 9.3 \cdot 10^{-24} \text{ Am}^2$$

and

$$k_B = 1.4 \cdot 10^{-23} \frac{\text{J}}{\text{K}}$$

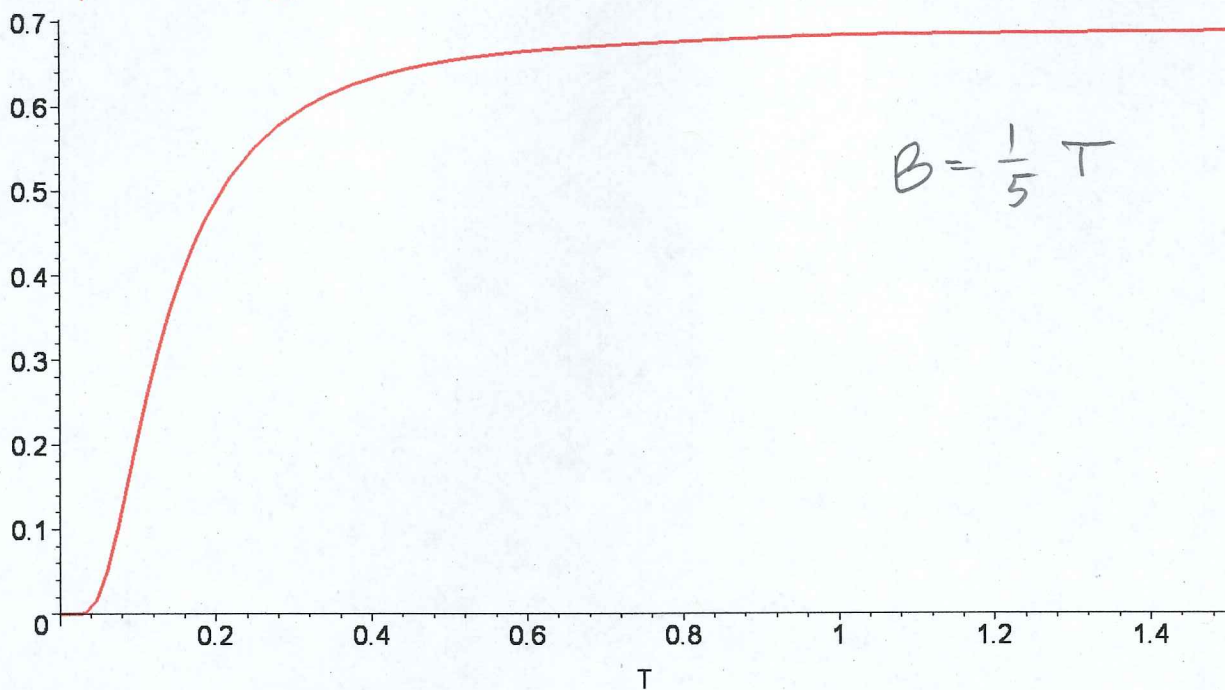
we get

$$S(T, N, B) = N \cdot (1.4 \cdot 10^{-23}) \left[\ln \left[2 \cosh \left(0.72 \frac{B}{T} \right) - 0.72 \frac{B}{T} \tanh \left(0.72 \frac{B}{T} \right) \right] \right]$$

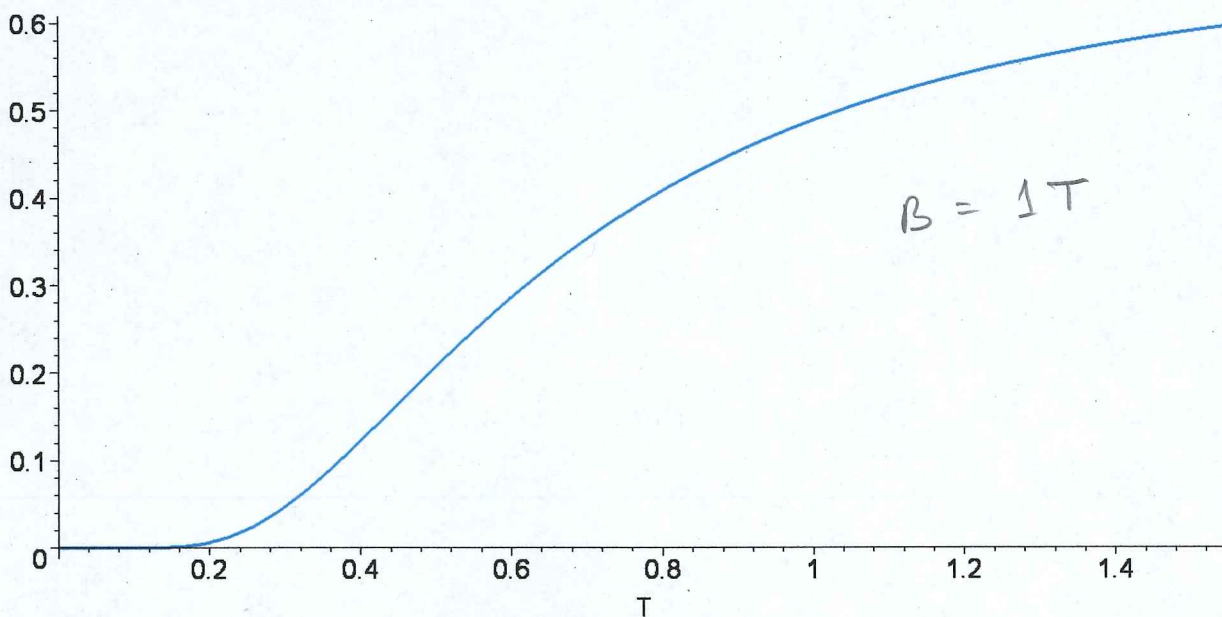
and so we can plot $S(T, N, B)$. I will

Take $N = \frac{1}{1.4} \cdot 10^{23}$ for convenience

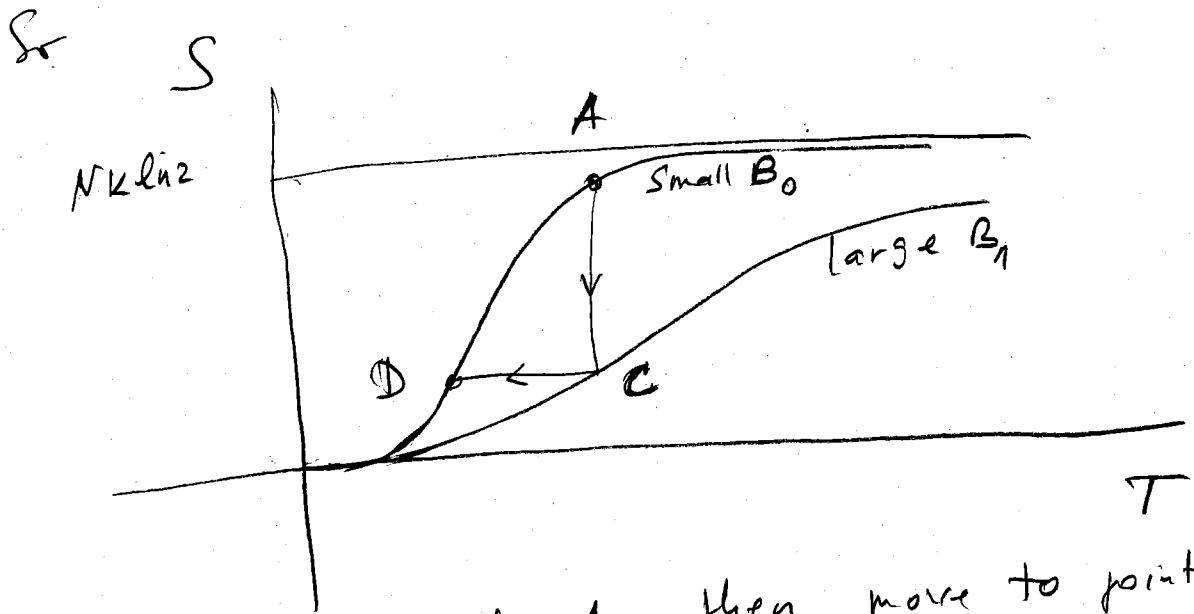
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> plot (subs (B=1/5, ln (2*cosh (0.72*B/T)) -0.72*B/T*tanh (0.72*B/T)) , T=0.  
.1.5, color=red);
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> plot (subs (B=1, ln (2*cosh (0.72*B/T)) -0.72*B/T*tanh (0.72*B/T)) , T=0..1  
.55, color=blue);
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When B is high the entropy grows less fast than when B is low.



If we start at A then move to point C and stop at D we lowered the temperature

$$T_D < T_A$$

Because the entropy depends on $\frac{B}{T}$ we can write from

$$S(T_A, N, B_A) = S(T_D, N, B_D)$$

that

$$\frac{B_A}{T_A} = \frac{B_D}{T_D}$$

$$\text{so } T_D = \frac{B_D}{B_A} T_A$$

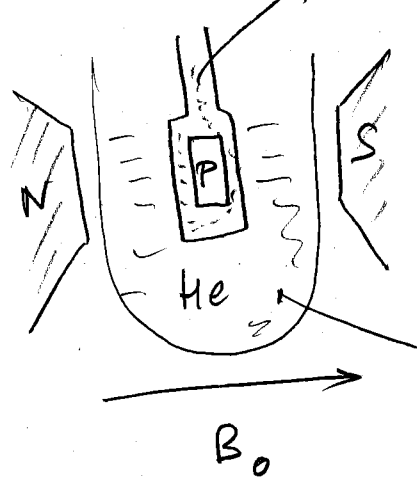
thus the magnetic field control the reduction of the temperature.

We will see below that from A to C the paramagnet is in contact with a heat bath at temperature $T_A = T_c$.

Suppose that we can turn OFF and ON the contact with the heat bath through a KNOB 1 and separately, we can turn OFF and ON the magnetic field through a KNOB 2. Playing with these KNOBS we can bring the paramagnetic salt at a temperature which is less than that of the heat bath. The heat bath can be He (Helium) at a temperature 1.13K. We can bring the paramagnetic sample at 0.73K (see the attached paper). This is an achievement because using only He (no B field) we cannot go beyond 1.13K (for the example given in the paper).

Process:

- (1) State A
 (T_A, B_0)
- Contact between P and He
 - B_0 is small, the magnet is OFF



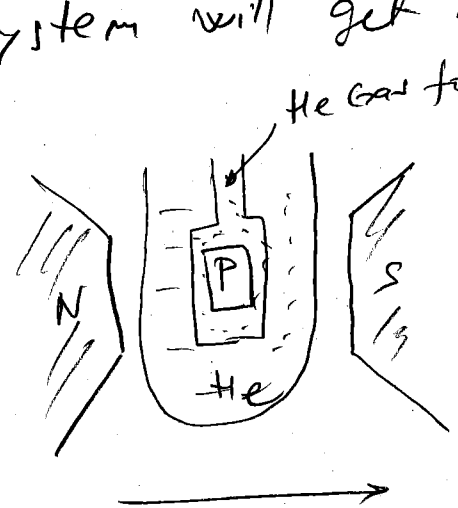
KNOB 1 = ON
 KNOB 2 = OFF

Liquid He
 (the heat bath)

Now turn on the KNOB2 and wait for some time. The system will get in the

(2) State G

- Contact between P and He so $T_C = T_A$
- B_1 is high

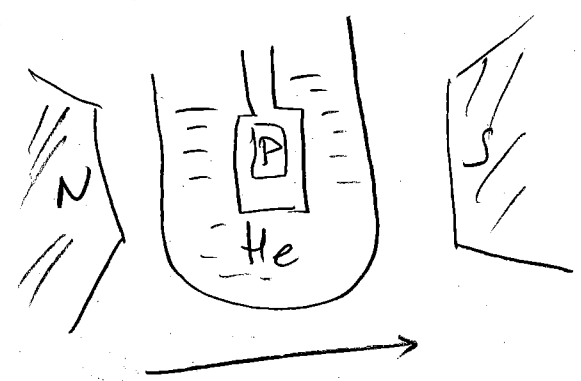


KNOB1 = ON
KNOB2 = ON

So far, because the contact with the He was on all the time, the paramagnete salt has the He temperature $T = 1.13 K$.
Now pump the He gas (not the liquid) and isolate the paramagnet from the liquid He.
Wait for some time.

(3) State D

- No contact between P and He ($T_D \neq T_A$)
- B_1 is high



KNOB1 = OFF
KNOB2 = ON

In the end

$$T_D < T_A$$

neglect of dissipation. We are here concerned with wave propagation in a good conducting medium. In the case of perpendicular magnetization one obtains two contributions to Q^{-1} arising separately from the two directions of circular polarization into which the cavity mode can be decomposed. In the case of parallel magnetization the final solution of the resonance problem is identical to that given by the usual theories, although demagnetizing factors for the oscillating fields are not specifically introduced. The results for R^2 are

$$R^2 = 2 \frac{\eta + (\eta^2 + 1)^{1/2}}{1 + (\eta\alpha T_2)^{-1}(\eta + 1)^{1/2}} \quad (\text{Bloch})$$

$$R^2 = 2 \frac{1 + \eta(\eta^2 + 1)^{-1/2}}{1 + (\alpha/\eta)^{1/2}(\eta + 1)^{1/2}} \quad (\text{LL})$$

where T_2 is the Bloch time constant and α is the coefficient of LL damping. We have used $T_2^{-1} = 3.5$ and 1.5×10^8 for nickel and Supermalloy, respectively, as determined by line width.

The agreement between measured and calculated values of R^2 , where calculations of the latter are based on wave propagation using Bloch damping, is good. There is also an improvement in line shape. Further details will be published in a subsequent paper.

We wish to thank Dr. S. O. Morgan and Mr. R. A. Chegwidzen of the Bell Telephone Laboratories for supplying the Supermalloy samples.

- ¹ C. Kittel, Phys. Rev. **71**, 270 (1947); **73**, 155 (1948).
² Kittel, Yager, and Merritt, Physica **15**, 256 (1949).
³ N. Bloembergen, Phys. Rev. **78**, 572 (1950).
⁴ D. Polder, Phil. Mag. **40**, 99 (1949).
⁵ Yager, Galt, Merritt, and Wood, Phys. Rev. **80**, 744 (1950).
⁶ C. L. Hogan, Bell System Tech. J. **31**, 1 (1952).
⁷ M. T. Weiss and A. G. Fox, Phys. Rev. **88**, 146 (1952).

Theory of Polarized Particles in Nuclear Reactions

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CONSIDER the most general two-body nuclear reaction,
 $a + X \rightarrow Y + b,$

where a and b have spins i and i' , respectively, and I and I' are the spins of X and Y . The density matrix for the initial state of a can always be written as a linear sum of its irreducible spin tensor moments $T_{\kappa}^{(a)}$, where q is the rank of the tensor and κ its component ($|\kappa| \leq q \leq 2i$). Similarly, the final states of b are determined by its spin tensor moments $T_{\kappa'}^{(b)}$ ($|\kappa'| \leq q' \leq 2i'$). In a previous communication¹ an expression was given for the spin tensor moments arising from a reaction initiated by an unpolarized beam; i.e., one in which all initial tensor moments other than $q = \kappa = 0$ vanish. This expression can now be generalized to the case of an arbitrarily polarized initial beam.

Let $T_{\kappa}^{(a)}$ be the expectation value of a spin tensor moment of b , (in channel α') measured with respect to the coordinate system with z axis along \mathbf{k}' and y axis along $\mathbf{k} \times \mathbf{k}'$, the normal to the scattering plane. The contribution to this quantity of an initial spin tensor moment $T_{\kappa}^{(a)}$ in channel α (measured relative to the incident beam direction) can then be written (since the effects of several initial tensor moments are additive):

$$T_{\kappa}^{(a')} = \frac{\lambda_{\alpha}^{\alpha'}(2i)}{4[(2i - q)(2i + q + 1)]^{1/2}(2I + 1)P_q([\frac{i}{i} + 1]^{1/2})} \\ \times \sum R^*(\alpha' i' s_1', \alpha' i' s_1'; J_1 \pi_1) R(\alpha i s_2, \alpha' i' s_2'; J_2 \pi_2) \\ \times W(i I q s_1; s_2) W(i' I' q' s_1'; s_2' i') (-1)^{\kappa} D_{\kappa, \kappa'}^{(L)}(\theta, \theta, 0) \\ \times G_{\kappa}^*(J_1 i_1 s_1; J_2 i_2 s_2; L q) G_{\kappa}(J_1 i_1' s_1'; J_2 i_2' s_2'; L q) T_{\kappa}^{(a)}, \quad (1)$$

where θ and ϕ are the direction angles of \mathbf{k}' relative to \mathbf{k} , D is the usual representation of the rotation group, W is the Racah function, and the sum is over L as well as all dynamical variables

$i_1 i_2 s_1 J_1 J_2 \pi_1$ and π_2 . The reaction matrix R is related to the scattering matrix S by $R = 1 - S$.

All selection rules are contained in the "geometrical" coefficient G , which is related to the X coefficient of Fano² by the relation

$$G_{\kappa}(J_1 i_1 s_1; J_2 i_2 s_2; L q) \\ = [2i_1 + 1)(2i_2 + 1)(2s_1 + 1)(2s_2 + 1)(2J_1 + 1)(2J_2 + 1)]^{1/2} i_1^{i_1 + 1/2} \\ \times \sum_r (2r + 1)^{1/2} (i_1 i_2 00 | i_1 i_2 r 0) (q r \kappa 0 | q r L \kappa) \\ \times X(J_1 i_1 s_1; J_2 i_2 s_2; L q r). \quad (2)$$

Note that the elements $(J_1 i_1 s_1)$, $(J_2 i_2 s_2)$, $(J_1 J_2 L)$, and $(s_1 s_2 q)$ of G must form a possible vector "triad." In addition, the elements $(i_1 i_2 L q)$ must form a possible vector "tetrad." The G coefficient is the generalization of the Z coefficient of Blatt and Biedenharn.³

The maximum complexity of the angular dependence is given by the largest value of L . This is given by the simultaneous conditions

$$L \leq \begin{cases} 2i_{\max} + q \\ 2i_{\max} + q - 1 & (\kappa = 0, q' \text{ even and } q \text{ odd}) \\ 2J_{\max} \\ 2i_{\max} + q' \\ 2i_{\max} + q' - 1 & (\kappa = 0, q \text{ even and } q' \text{ odd}) \end{cases} \quad (3)$$

along with the condition that L must be even if $\kappa(\kappa')$ is zero, $q(q')$ is even and the interfering levels have the same parity. These rules are more restrictive than those given previously.⁴

A more detailed paper, which is in preparation, will give the derivation of the above results as well as applications to arbitrarily polarized target nuclei, reactions involving gamma-rays, and the detection of polarized particles. In addition, an alternative expression for G which is very convenient for tabulation will be given. It is a pleasure to thank Dr. T. A. Welton for many stimulating discussions.

¹ A. Simon, Phys. Rev. **90**, 325 (1953). See also A. Simon and T. A. Welton, Phys. Rev. (to be published).

² U. Fano, National Bureau of Standards Report 1214, p. 48 (unpublished).

³ J. M. Blatt and L. C. Biedenharn, Revs. Modern Phys. **24**, 258 (1952).

⁴ L. Wolfenstein, Phys. Rev. **75**, 1664 (1949).

Cyclic Adiabatic Demagnetization*

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CYCLIC demagnetization apparatus has been devised and used for maintaining the temperature of a paramagnetic salt sample at 0.73°K while the heat sink temperature was 1.13°K. The refrigerant is a second salt sample 12 cm distant, movable within an evacuated case and capable of being forced into contact either with the surrounding vacuum case which is immersed in pumped liquid helium or the first-named salt sample which is to be cooled.

A sectional view of the apparatus is shown in Fig. 1. The two salt samples are placed at a distance from each other so that a magnetic field may be applied to one sample only at a time. Temperatures are estimated from susceptibility measurements. The top sample can be brought into thermal contact with the case at its upper end or with a tubular extension of the bottom sample at its lower end. Iron ammonium alum is the salt used in this experiment. It is sealed in a brass capsule with a small amount of helium as the heat transfer agent.

In operation the poles of a permanent magnet (1850 gauss) are brought up to the top sample while it is in contact with the helium bath and kept there for a period of two minutes. The contact is broken and the magnet removed. The demagnetized sample, which is now cold, is placed in contact with the lower sample. At the end of four or five minutes substantial equalization of temperatures has been achieved, and a new cycle is started. After several cycles the lower sample acquires an equilibrium

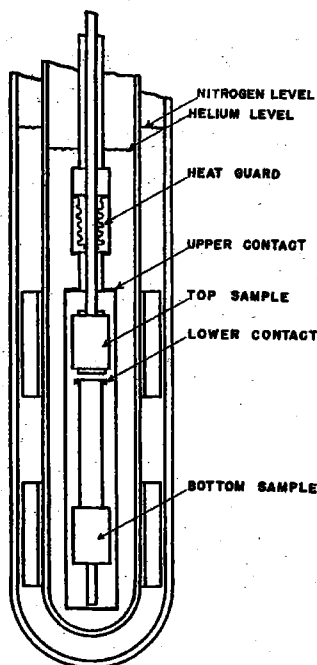


FIG. 1. Cyclic demagnetization system.

temperature which is determined principally by the magnitude of the heat leaks and the strength of the magnet.

Heat generated in the system by mechanical vibration seems to be much more important than thermal conduction of the supporting members. It is also more elusive with respect to control measures.

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Energy Band Structure in Silicon Crystal

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THE energy band structure in silicon has been calculated in a way quite similar to that used by Von der Lage and Bethe¹ for sodium. The levels evaluated are the lowest three having the wave vector $k=0$. Angular functions were constructed from the Von der Lage-Bethe cubic harmonics, and radial functions were computed numerically up to $l=6$, making use of the effective nuclear charge given in Mullaney's paper.² Results are shown in Table I, with the results obtained by Mullaney² and Holmes.³

TABLE I. Energy values for the lowest three levels at $k=0$, in ev.

Type of wave function	Our results	Mullaney's results	Holmes' results
$(\alpha+\beta)A^a$	0.405	very high	4.24
$(\delta+\epsilon)S^b$	-0.675	-3.24	-5.72
$(\alpha+\beta)A^c$	-15.8	-18.71	...

^a Bottom of conduction band.

^b Top of filled band.

^c Bottom of filled band.

The notation α , α' , β , β' , ... denotes the type of cubic harmonics after Von der Lage and Bethe.¹ The point group for the diamond type lattice is the tetrahedral group which is a subgroup of the cubic group. Therefore, function subspaces of this group are com-

bined into the five types $(\alpha+\beta)$, $(\alpha'+\beta')$, $(\gamma+\gamma')$, $(\delta+\epsilon)$, $(\delta'+\epsilon')$. The symmetric and antisymmetric properties of the wave functions for a translation to an adjacent atomic cell polyhedron following an inversion operation are indicated by the suffixes S and A . Function subspaces of the space group for $k=0$ are split into the types $(\alpha+\beta)_S$, $(\alpha+\beta)_A$, ... For instance, the $(\alpha+\beta)_S$ type wave function is expanded in terms of α - and β -type harmonics, and has the same form in the adjacent polyhedron referring to inverted coordinate axes. As boundary conditions, the expansion coefficients are calculated according to the symmetry property and Bloch periodicity, and fitting is performed at the center and on a circle which is an intersection with the equal volume sphere, on the hexagonal face of the atomic polyhedron, just as in the case treated by Von der Lage and Bethe.

It seems from our results that the more precise method gives better agreement with the experimental data. (See Table II.)

TABLE II. Comparison of results of calculations with experimental data.

	Our results	Mullaney	Holmes	Experiment
Energy gap of band in ev	1.08	very large	9.96	1.1
Band width in ev	15.1	15.5	...	19.2

[In fact, the $(\delta+\epsilon)_S$ type wave function includes contributions from $l=3$ and $=4$ of the same order as those from $l=1$ and $=2$.] Therefore, from this agreement, the highest point of the filled band and lowest point of the conduction band presumably occur at the center of the Brillouin zone.

¹ F. C. Von der Lage and H. A. Bethe, Phys. Rev. 71, 612 (1947).

² J. F. Mullaney, Phys. Rev. 66, 326 (1944).

³ D. K. Holmes, Phys. Rev. 87, 782 (1952).

The Phosphorescence of Thoria*

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WHILE observing luminescent effects resulting from the irradiation of various metallic oxides with nuclear particles, the writers chanced to note a continuous light emission from thoria. This luminescence was detected prior to application of any primary excitants such as alpha-particles, beta-rays, gamma-rays, or ultraviolet light. The property is exhibited by both Norton's reddish arc-melted coarsely crystalline thoria and white pieces of thoria sintered from powdered ThO_2 .^{1,2} The measurements of light intensity were performed with the use of phototubes RCA-5819 and RCA-1P28 as detectors.

Owing to the natural radioactivity of Th^{232} and its daughter elements, ThO_2 emits $\sim 22,000$ α 's per gram per second, $\sim 15,000$ β 's per gram per sec, and $\sim 20,000$ γ 's per gram per sec. These internally emitted nuclear radiations give rise through self-absorption in the thoria to the observed luminescence which has apparently attained a nearly constant intensity, decaying with the enormous half-period of Th^{232} . When a small piece of crystalline thoria was placed before the slit of a small Hilger quartz spectrograph, no noticeable darkening of the photographic plate was produced after an exposure of two weeks. However, when the

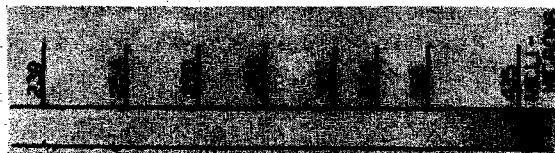


FIG. 1. Luminescent emission of thoria under alpha-irradiation.