How much entropy reduction does evolution require?

Emory F. Bunn*

Physics Department, University of Richmond

(Dated: December 11, 2008)

a. Introduction. In a recent article in this journal,[1] Daniel Styer addresses the claim, often made by creationists, that evolution requires a decrease in entropy in conflict with the second law of thermodynamics. He correctly explains that this claim rests on misunderstandings about the nature of entropy and the second law. In particular, the Earth is constantly absorbing sunlight and radiating energy into space, resulting in an enormous increase in entropy, which can counteract the decrease presumed to be required for evolution. This argument is known to scientists who participate in the evolution-creationism debate, but it is invariably described only in a general, qualitative way. Styer's article attempts to fill this gap with a quantitative argument. Because the incorrect claim is widespread among creationists, a detailed, quantitative rebuttal is an extremely good idea.

Styer computes the entropy increase due to solar radiation and compares it with an estimate of the entropy decrease required for evolution. Unfortunately, his argument rests on an unjustified and probably incorrect assumption about the latter quantity. I will present a modified version of the argument, which does not depend on this assumption and which shows that the final conclusion remains valid: the entropy decrease required for evolution is orders of magnitude too small to conflict with the second law of thermodynamics.

Styer correctly computes the rate of entropy change due to the flow of solar energy through the Earth, finding it to be

$$\frac{dS}{dt} = +4 \times 10^{14}\,(\mathrm{J/K})/\mathrm{s} = +(3 \times 10^{37} k)\,\mathrm{s}^{-1}$$

where k is Boltzmann's constant. There is a problem with the second law of thermodynamics only if evolution requires an entropy decrease of at least this much. To show that this is not the case, Styer estimates the rate of evolutionary entropy decrease, based on the following assumption:

> Suppose that, due to evolution, each individual organism is 1000 times "more improbable" than the corresponding individual was 100 years ago. In other words, if Ω_i is the number of microstates consistent with the specification of an organism 100 years ago, and Ω_f is the number of microstates consistent with the specification of today's

"improved and less probable" organism, then $\Omega_f = 10^{-3}\Omega_i$. I regard this as a very generous rate of evolution, but you may make your own assumption.

Far from being generous, a "probability ratio" of $\Omega_i/\Omega_f = 10^3$ is far too low. One of the central ideas of statistical mechanics is that practically anything you can imagine doing to a macroscopic object (say, one as large as a cell) results in an exponentially large change in the multiplicity (i.e., the number of accessible microstates). In part (b) I will provide some order-of-magnitude illustrations of this, and in part (c) I will present a modification of Styer's argument that does not depend on this assumption.

b. Rough estimates. Let us consider an example of about the smallest evolutionary change imaginable in an organism. Suppose that the only difference between the organism 100 years ago and its descendent today is that somewhere inside of one cell, one additional protein molecule has been formed. Suppose further that the amino acids that make up the protein were already present in the corresponding cell of the ancestor, so that all that had to happen was for them to be assembled in the right order.

We can crudely estimate the entropic cost of this change by imagining assembling the protein one amino acid at a time. At each step, we must take a molecule that was freely moving through the cell and place it in a specific place. If the amino acids were previously in a dilute solution in the cell, then the multiplicity loss due to this process is approximately n_Q/n, where n is the number density of amino acids and n_Q is the density at which the amino acids would reach quantum degeneracy.[1] This is certainly a large factor: amino acids in a cell are far from degenerate. To assemble a protein with N_a amino acids, we would repeat this process $N_a - 1$ times, resulting in the exponentially large number $\Omega_i/\Omega_f \sim (n_Q/n)^{N_a-1}$. For instance, if $n_Q/n = 10$ (surely far too low) and $N_a = 20$, the multiplicity ratio is $\sim 10^{19}$ for even this minute alteration of the organism.

This number results from creating one protein molecule. If instead a single gene turned on in the cell, and produced N_p copies of the protein, the above multiplicity ratio would be raised to the N_p power.

The above estimate is of course extremely rough. For example, it neglects the internal degrees of freedom of the protein (which are surely far fewer than those of the free amino acids), and entropy changes due to the energy absorbed or emitted during the formation of chemical bonds.

[1] To see this, imagine that there are k amino acids in solution, with N available quantum states. Nondegeneracy means that $N \gg k$. The multiplicity is $\Omega(k) = \binom{N}{k}$. Taking one molecule out of solution causes the multiplicity to go down by a factor $\Omega(k)/\Omega(k-1) = (N-k+1)/k \approx N/k = n_Q/n$.

To include the latter, we can simply note that the multiplicity change associated with a chemical reaction is $e^{\mu/kT}$. For a process involving a small number of molecules, the chemical potential μ (also known as the Gibbs free energy per particle) is generically of order eV or more, implying multiplicity changes of order $e^{40} \approx 10^{17}$ at biological temperatures.

The actual differences due to evolution will be much more complicated than the above scenario, but they will be built out of many such ingredients, and the resulting multiplicity changes will be at least this large. This sort of number is the ante to enter this particular game.[2]

This underestimation of the required entropy reduction for evolution undermines Styer's argument. To fix the problem, we should estimate this quantity in a way that is certain to be an overestimate. I will provide such an argument in the next section.

c. A more robust argument. Let us estimate the amount of entropy reduction involved in the evolution of life on Earth, taking care to make assumptions that overestimate the requirement. Consider the entropy difference between two systems: Earth as it actually is at the present moment, and a hypothetical Dead-Earth on which life never evolved. We will assume that every atom in Earth's biomass is located in Dead-Earth's atmosphere in its simplest molecular form, and that Dead-Earth and Earth are otherwise identical. Furthermore, when considering the entropy of Earth, we will assign *zero* entropy to the biomass. That is, we will imagine that, in order to produce life on Earth, it is necessary to put every atom in the biomass into its exact present-day quantum state. These assumptions maximize the entropy of Dead-Earth and minimize that of Earth, so the difference between the two entropies grossly overestimates the required entropy reduction for the production of life in its present form.

With these assumptions, we can estimate the entropy difference as

$$\Delta S_{\rm life} = S_{\rm Dead\ Earth} - S_{\rm Earth} \approx -N_b \mu / T,$$

where N_b is the number of molecules in the biomass and μ is a typical chemical potential for a molecule in the atmosphere. Using standard formulae for an ideal gas (e.g., equation 3.63 of the book by Schroeder[2]), we find $\mu/kT \sim 10$. The total carbon biomass of Earth has been estimated[3] at $\sim 10^{15}$ kg. Even if we scale this value up by a generous factor of 100 to account for other elements, we still have fewer than 10^{43} molecules. So the entropy reduction required for life on

[2] It is of course logically possible (although implausible) that many such changes, some thermodynamically favorable and some unfavorable, could be extremely fine-tuned to give a small multiplicity change, but we are certainly not allowed to assume such fine-tuning for the purposes of the argument at hand.

Earth is (far) less than

$$\Delta S_{\rm life} \sim 10^{44} k.$$

Comparing this with the rate of entropy production due to sunlight, we find that the second law is satisfied as long as the time required for life to evolve on Earth is at least

$$\Delta t = \frac{\Delta S_{\rm life}}{dS/dt} \sim 10^7\,{\rm s},$$

or less than a year. Life on Earth actually took four billion years to evolve, so the second law of thermodynamics is safe[3].

Acknowlegment. I thank Andrew Bunn for helpful comments.

---

* Electronic address: ebunn@richmond.edu

[1] D. F. Styer, Am. J. Phys **76**, 1031 (2008).

[2] D. V. Schroeder, *An Introduction to Thermal Physics* (Addison Wesley Longman, 2000).

[3] W. B. Whitman, D. C. Coleman, and W. J. Wiebe, Proc. Natl. Acad. Sci. USA **95**, 6578 (1998).

[3] Although creating all of life in six days might be thermodynamically problematic.