Galactic Spring Cleaning

Searching for Dust Contamination of CMB Maps Using Wavelet Transforms

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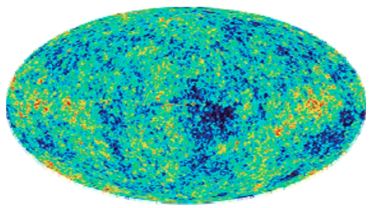
Background Information

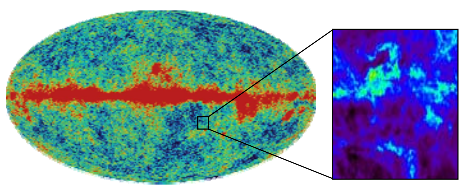
Our group works with data from the Wilson Microwave Anisotropy Probe (**WMAP)** with a current main goal of searching for non-Gaussianity in the Cosmic Microwave Background (CMB). The leading theory of the early universe is inflation, which predicts a Gaussian CMB; i.e. the CMB temperature fluctuations contain no information beyond the power spectrum. Investigating potential non-Gaussianity is then, in a way, putting inflationary theory to the test!



Our Big Questions

Notice the image below looks the same as the WMAP data set above, except for some red (i.e. "hot") structure through the middle. This structure is foreground contamination from Galactic dust, among other sources.

If we zoom in and consider a small patch of the sky, and remove / "ignore" the CMB fluctuations, we are left with a small patch of only foreground (dust).



Schlegel, Finkbeiner, Davis (1998)

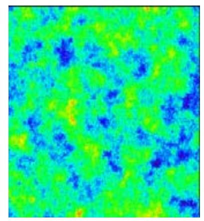
Now, let's suppose the CMB is Gaussian. The problem is that Galactic dust (even in "small" amounts) in the foreground of WMAP's view can make the CMB *appear* to be non-Gaussian! Searches for CMB non-Gaussianity therefore require very good levels of dust contamination and diagnostics of residual contamination. Our question, then, is:

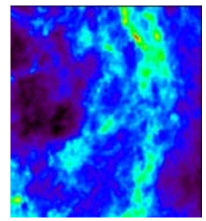
Is low-level dust contamination in a CMB map easier to detect after filtering the map in a wavelet-transformed basis?

The approach we use to answer this question is the following: First we simulate maps that are contaminated with dust. We do this by taking a piece from the Schlegel, Finkbeiner, Davis (SFD) data set ,which represents dust, and add it to a purely Gaussian map. We then apply our test to a map with and without a wavelet transform. We have used a variety of statistics in the past; we focus here on the mean square test (M) defined below. We then compare the M values to see which is better. The reason we simulate the dust maps as opposed to using raw data to find contamination is because we can check our answer. We decide how much dust contamination there is and then we see if our test can detect it.

The Discrete Wavelet Transform (DWT)

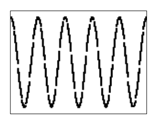
Gaussian map **Dust maps**



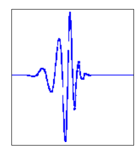


Perhaps the simplest way to think of a wavelet transform is by analogy: consider it a Fourier transform that uses wavelet functions instead of sine and cosine functions.

We are interested in the DWT because wavelets are functions localized in space, much like the often filamentary structure of dust patches.



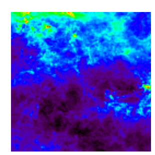
Fourier Transforms use Non localized functions with different amplitudes and wavelengths



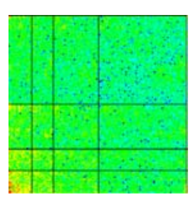
Wavelet Transforms use localized functions with different amplitudes, wavelengths, and positions.

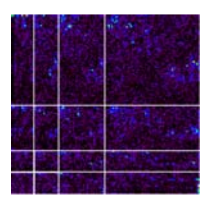
We find that **a very high percentage of power can be retained by keeping very few wavelet coefficients**.

Normalizing the DWT



We start with a typical dust map (top), and then we take a DWT (left: amplitude in log scale). The DWT splits the image into distinct blocks by the scale of the wavelength. The top right block is the smallest scale and the bottom left is the biggest. Before we normalize it the largest scaled wavelengths receive the most weight in the DWT. For some tests, we want to give all wavelet scales equal weight, so we compute the *normalized DWT* by normalizing each block to have the same variance (right).

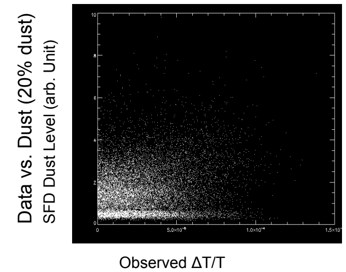




Mean Square Test (M)

The mean square test compares mean square values of places with high dust with mean square values of places with low dust. This test works by choosing a cutoff for the value of the dust, then comparing the ratio of mean square values above and below the cutoff. For uncontaminated maps, the ratio should be 1, but for maps with dust, the mean square value above the cutoff will be greater than the value below.

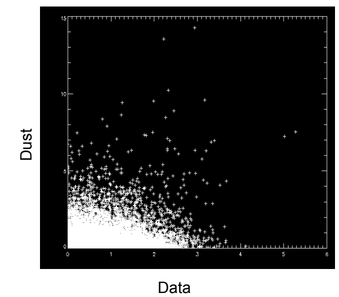
$$M = \frac{\langle T^2{}_{Dust>cutoff} \rangle}{\langle T^2{}_{Dust<cutoff} \rangle}$$



Mean Square Test with Wavelets

If we take a normalized DWT of our data and then apply the mean square test we hypothesize that the power of the test, the probability at which we can see contamination at a 95% confidence, would be greater than with no wavelet transform. Since dust is efficiently represented by only a few wavelets we can be sure that wavelets of high dust in the sample map will correspond to areas of high dust in the dust map.

Normalized Wavelet Transform Data vs. Dust (20%dust)



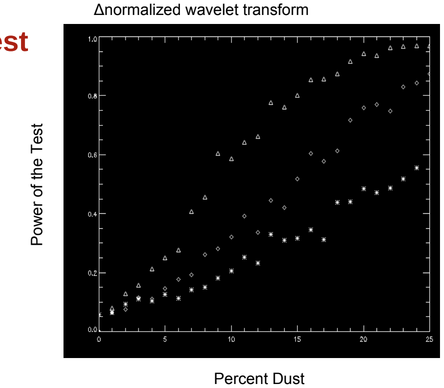
Power of Test

Since our test's practical application is to tell whether or not all the dust has been removed from a map we apply our test to Gaussian maps that have different percentages of dust contamination and see at what certainty we can say that the dust has been removed.

Dust percentage vs. Power of the test at 50% cutoff

*mean square test ◊ with wavelet transform

Δnormalized wavelet transform



Results

We found that for all cutoffs the power of the test was greater for maps with a normalized wavelet transform than for maps with no wavelet transform. For some cutoffs the no wavelet transform did better than the wavelet transform.

References

Bennett et al., Ap. J. Supp., 148, 1 (2003).

Bennett et al., Ap. J. Supp., 148, 97 (2003).

Genovese, Wavelets, Encyclopedia of Statistical Sciences, Kotz & Johnson, eds. (Wiley, 1999).

Schlegel, Finkbeiner, & Davis, Ap. J., 500, 525 (1998).

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