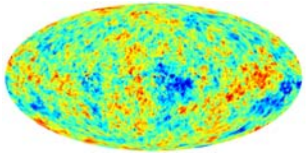
Discrepancies Between Large-Scale Microwave Background Fluctuations and Theoretical Predictions

Austin Bourdon, E.F. Bunn

Physics Department, University of Richmond, VA 23173

Background

The Cosmic Microwave Background (CMB) contains subtle temperature variations that have been characterized in detail by the Wilson Microwave Anisotropy Probe. Variations in temperature are represented in the picture below with blue corresponding to lower than average temperatures and red corresponding to higher than average temperatures. The small temperature fluctuations shed light on how galaxies and the rest of the universe were formed.



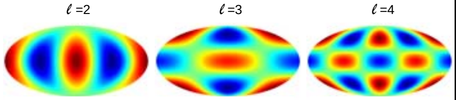
In almost all respects, the WMAP data are consistent with standard theories of the early universe, particularly the inflationary theory. However, some discrepancies have apparently been found in the large-angular-scale properties of the maps. The statistical significance of these discrepancies is disputed.

We have examined a class of proposed explanations for these anomalies. We show that, contrary to popular belief, these explanations worsen rather than improve the consistency between data and theory and therefore cannot be accepted as solutions to this problem.

Spherical Harmonics

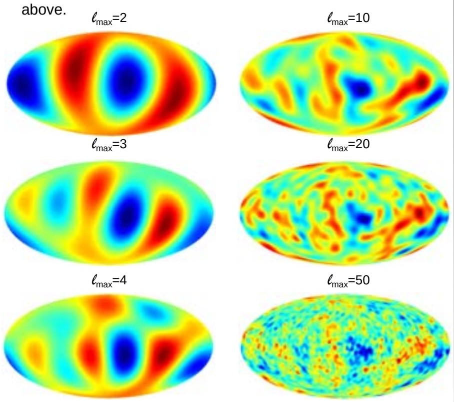
Spherical harmonics provide an important quantitative tool for studying the CMB. Overall, spherical harmonics make it possible to study structures in a map at different scales. Values of ℓ are assigned to different wavelengths—low values of ℓ correspond to long wavelengths while high values correspond to short wavelengths. The pictures below illustrate the different spherical harmonics for ℓ = 2, 3, and 4.

ℓ =2 ℓ =3 ℓ =4



WMAP Data Spherical Harmonics

The CMB data are represented as a sum of all of the spherical harmonics with different amplitudes. Below are displayed the spherical harmonics for the real WMAP data. Each map shows the harmonics from ℓ =2 (the smallest value allowed) to some maximum ℓ_{max}. As the values of ℓ_{max} increase, smaller features appear on the map. Values of ℓ higher than 50 would appear more similar to the real map above.



Low Amplitude of Large Scale Wavelengths

One anomaly in the WMAP data is a lower amplitude of large scale wavelengths than expected in theoretical models. This lack of large scale power can be quantified by the quadrupole power, which is the amplitude of the ℓ =2 spherical harmonics, or by the $S_{1/2}$ statistic defined below.

Neither the low quadrupole nor the $S_{1/2}$ statistic calculated from the WMAP data fall within their predicted ranges, pointing to potential flaws in the data. There have been claims stating that eccentricity and cosmic dust contamination may be responsible for the lack of large scale power. This research shows that explanations of this sort actually reduce the probability of getting such low values. That is, they make the problem worse, rather than solving it.

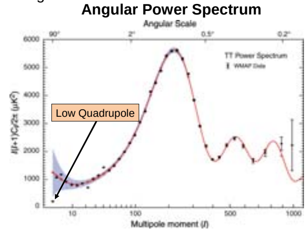
The Low Quadrupole

The low quadrupole value provides the simplest way to quantify the lack of large scale power in the CMB. It is illustrated by the angular power spectrum of the WMAP data. Basically, the angular power spectrum is a representation of the amplitude of the temperature variations within the map as a function of angular scale.

Shown below is the power spectrum of the data returned by WMAP. Along the x axis are the spherical harmonic ℓ values while along the y axis are the temperature variations. The quadrupole corresponds to ℓ =2 (the largest angular scale observable).

Note that the quadrupole power C_2 lies outside of the theoretically predicted range.

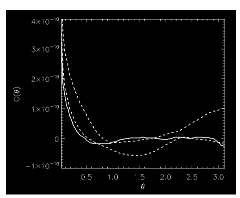
Angular Power Spectrum



Two-point Correlation Function

The two point correlation function is necessary for calculating the $S_{1/2}$ statistic. Overall, this function determines how closely points on a map relate to one another. It separates two points by an angle θ and then calculates the average of the products of the temperature at those points:

$$C(\theta) = \left\langle T(P_1)\, T(P_2) \right\rangle_{P_1, P_2 \text{ separated by } \theta}$$



In this graph, the correlation function values for the KP0 mask cut of the WMAP data is shown plotted over the theoretical range of correlation (represented by the two dotted lines). The graph's x-axis corresponds to the angle of separation between the two points.

From the above graph, it is evident that the correlation function values for the real WMAP data do not follow the theoretical path. Instead, the correlation function remains relatively flat around zero. Such flatness is characteristic of a universe with a lack of power in long-wavelength modes.

The $S_{1/2}$ statistic is defined to be the integral of $C^2(\theta)$ over all angles from 60º to 180º.

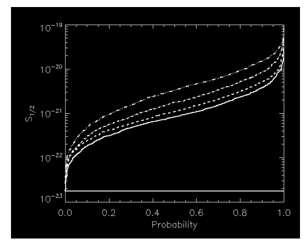
$$S_{1/2} = \int_{\pi/3}^{\pi} C^2(\theta) \sin(\theta)\, d\theta$$

Since the correlation function remains near zero over this range, the value of $S_{1/2}$ for the real data is significantly smaller than predicted in theoretical models. As noted above, various explanations have been proposed to explain this discrepancy.

$S_{1/2}$ Probability and Eccentricity

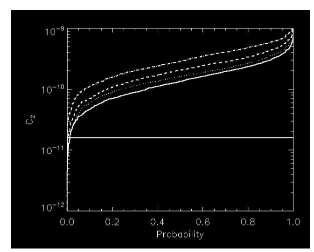
Eccentricity is basically a measure of ellipticity in the universe. If the universe were expanding faster in one direction than the other, this would result in an elliptical universe with an eccentricity greater than zero.

If eccentricity is in fact causing the number of low wavelengths to decrease, then the probability that the $S_{1/2}$ values are less than the value for the real data should be greater in models with eccentricity than in those without. The probability graph below shows that precisely the opposite is true. The solid curve is the cumulative probability distribution for the standard (no-eccentricity) model, while the other curves from bottom to top correspond to eccentricities .005, .0062, .0074. The horizontal line represents the value for the real data.



From the graph, it evident that $S_{1/2}$ values are raised, not lowered, and the probability of attaining values below the horizontal line is decreased by the addition of eccentricity.

Quadrupole and Eccentricity



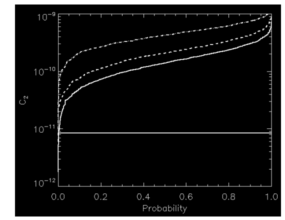
The above graph shows the probabilities of getting particular values of the quadrupole power C_2 for maps with a KP0 mask cut. The four curves in the graph correspond to same eccentricity values represented in the $S_{1/2}$ eccentricity graph.

The results show that adding eccentricity to the CMB map would decrease the probability of finding C_2 values lower than the real data.

Adding a Spot to the Map to Test Low Quadrupole

In order to test the hypothesis that adding contaminants such as cosmic dust to a map would decrease the probability of attaining a lower quadrupole value, simulation CMB maps were created that added non-Gaussian contamination spots. These spots are areas of the map with a constant amplitude of $\Delta T/T$ = .00005 and a set radius.

Graphed below are curves that represent the probabilities of attaining different C_2 values.



The three curves each correspond to contamination spots of different sizes: 0 radians, .5 radians, and 1 radian. The horizontal line in the graph represents the real C2 value obtained from the WMAP data.

From this graph, it is evident that higher contamination levels increase the C_2 values of the power spectrum, decreasing the probability of attaining a value of C_2 below the WMAP calculation.

Conclusion

In all simulations, the odds of finding a low $S_{1/2}$ or quadrupole value were never high and were decreased by adding eccentricity and contamination. There is some controversy in the field over the statistical significance of the lack of large-scale power in the CMB. Our results show that, if there is a problem, then contaminants such as eccentricity or foregrounds cannot solve the problem; in fact, they make it worse.

References

Wilkinson Microwave Anisotropy Probe, "New Three Year Results on Oldest Light in the Universe" http://map.gsfc.nasa.gov/m_mm.html, viewed July 16, 2007.

L. Campanelli, P. Cea, and L. Tedesco, "Ellipsoidal Universe Can Solve the CMB Quadrupole Problem", Physical Review Letters, 97, 131302 (2007).

Amir Hajian, "On the Apparent Lack of Power in the CMB Anisotropy at Large Angular Scales", arXiv:astro-ph/0702723v1.

G. Hinshaw et al., "Three-Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Temperature Analysis", Astrophysical Journal Supplement, 170, 288 (2007).

This work was supported by NSF grant 0507395.