Discrepancies Between Large-Scale Microwave Background Fluctuations and Theoretical Predictions

Austin Bourdon, E. F. Bunn
Physics Department, University of Richmond, VA 23173

Background

The Cosmic Microwave Background (CMB) contains subtle temperature variations that have been characterized in detail by the Wilson Microwave Anisotropy Probe. Variations in temperature are represented in the picture below with blue corresponding to lower than average temperatures and red corresponding to higher than average temperatures. The small temperature fluctuations shed light on how galaxies and the rest of the universe were formed.

In almost all respects, the WMAP data are consistent with standard theories of the early universe, particularly the inflationary theory. However, some discrepancies have apparently been found in the large-angular-scale properties of the maps. The statistical significance of these discrepancies is disputed.

We have examined a class of proposed explanations for these anomalies. We show that, contrary to popular belief, these explanations worsen rather than improve the consistency between data and theory and therefore cannot be accepted as solutions to this problem.

Spherical Harmonics

Spherical harmonics provide an important quantitative tool for studying the CMB. Overall, spherical harmonics make it possible to study structures in a map at different scales. Values of $l$ are assigned to different wavelengths—low values of $l$ correspond to long wavelengths while high values correspond to short wavelengths. The pictures below illustrate the different spherical harmonics for $l = 2, 3,$ and $4$.

WMAP Data Spherical Harmonics

The CMB data are represented as a sum of all of the spherical harmonics with different amplitudes. Below are the displayed spherical harmonics for the real WMAP data. Each map shows the harmonics from $l = 2$ (the smallest value allowed) to some maximum $l_{max}$. As the values of $l_{max}$ increase, smaller features appear on the map. Values of $l$ higher than 50 would appear more similar to the real map above.

Low Amplitude of Large Scale Wavelengths

One anomaly in the WMAP data is a lower amplitude of large scale wavelengths than expected in theoretical models. This lack of large scale power can be quantified by the quadrupole moment of the power spectrum. The angular power spectrum is a representation of the amplitude of the temperature variations within the map as a function of angular scale.

The Low Quadrupole

The low quadrupole value provides the simplest way to quantify the lack of large scale power in the CMB. It is illustrated by the angular power spectrum of the WMAP data. Basically, the angular power spectrum is a representation of the amplitude of the temperature variations within the map as a function of angular scale.

From this graph, it is evident that higher contamination levels increase the $C_2$ values of the power spectrum, decreasing the probability of attaining a value of $C_2$ below the WMAP calculation.

The three curves each correspond to contamination spots of different sizes: 0 radians, .5 radians, and 1 radian. The horizontal line in the graph represents the real $C_2$ value obtained from the WMAP data.

From this graph, it is evident that higher contamination levels increase the $C_2$ values of the power spectrum, decreasing the probability of attaining a value of $C_2$ below the WMAP calculation.

Conclusion

In all simulations, the odds of finding a low $S_{12}$ or quadrupole value were never high and were decreased by adding eccentricity and contamination. There is some controversy in the field over the statistical significance of the lack of large-scale power in the CMB. Our results show that, if there is a problem, then contaminants such as eccentricity or foregrounds cannot solve the problem; in fact, they make it worse.

References

This work was supported by NSF grant 0507395.

The two point correlation function is necessary for calculating the $S_{12}$ statistic. Overall, this function determines how closely points on a map relate to one another. It separates two points by an angle $\theta$ and then calculates the average of the products of the temperature at those points:

$$C(\theta) = \langle (T(P_1) T(P_2)) \rangle_{(P_1, P_2) \text{ separated by } \theta}$$

In this graph, the correlation function values for the KP0 mask cut of the WMAP data is shown plotted over the theoretical range of correlation (represented by the two dotted lines). The graph's x-axis corresponds to the angle of separation between the two points.

From the above graph, it is evident that the correlation function values for the real WMAP data do not follow the theoretical path. Instead, the correlation function remains relatively flat around zero. Such flatness is characteristic of a universe with a lack of power in long-wavelength modes.

The $S_{12}$ statistic is defined to be the integral of $C(\theta)$ over all angles from 60° to 180°.

$$S_{12} = \int_0^{\pi/2} C(\theta) \sin(\theta) d\theta$$

Since the correlation function remains near zero over this range, the value of $S_{12}$ for the real data is significantly smaller than predicted in theoretical models. As noted above, various explanations have been proposed to explain this discrepancy.

The above graph shows the probabilities of getting particular values of the quadrupole power $C_2$ for maps with a KPO mask cut. The four curves in the graph correspond to same eccentricity values represented in the $S_{12}$ eccentricity graph.

The results show that adding eccentricity to the CMB map would decrease the probability of finding $C_2$ values lower than the real data.