Homework 29

1. $\int t^3(t^4-4)^2\,dt \quad u=t^4-4 \quad \frac{du}{dt}=4t^3 \quad dt=\frac{du}{4t^3}$

$\int \cancel{t^3}(u)^2\cdot\frac{du}{4\cancel{t^3}} = \frac{1}{4}\int u^2\,du = \frac{1}{4}\cdot\frac{u^3}{3} = \boxed{\frac{1}{4}\cdot\frac{(t^4-4)^3}{3}+C}$

2. $\int 7x^5\sin(x^6)\,dx \quad u=x^6 \quad \frac{du}{dx}=6x^5 \quad dx=\frac{du}{6x^5}$

$\int 7\cancel{x^5}\sin(u)\frac{du}{6\cancel{x^5}} = \frac{7}{6}\int\sin(u)\,du = \frac{7}{6}\cdot -\cos(u) = \frac{7}{6}\cdot -\cos(x^6) = \boxed{-\frac{7\cos(x^6)}{6}+C}$

3. $\int \frac{\ln^8(z)}{z}\,dz = \boxed{\frac{\ln^9(z)}{9}+C}$

4. $\int \frac{e^{5x}}{1+e^{5x}} \quad u=1+e^{5x} \quad \frac{du}{dx}=5e^{5x} \quad dx=\frac{du}{5e^{5x}}$

$\int \frac{\cancel{e^{5x}}}{u}\cdot\frac{du}{5\cancel{e^{5x}}} = \frac{1}{5}\int\frac{1}{u}du = \frac{1}{5}\cdot\ln(u) = \boxed{\frac{1}{5}\cdot\ln(1+e^{5x})+C}$

☆ 5. $\int \frac{2e^{4\sqrt{y}}}{\sqrt{y}}\,dy \quad u=4\sqrt{y} \quad \frac{du}{dy}=\frac{2}{\sqrt{y}} \quad du=\frac{2dy}{\sqrt{y}} \quad \sqrt{y}\,du=2dy \quad dy=\frac{\sqrt{y}}{2}$?

$2\int\frac{e^u}{2}du = 2\cdot\frac{1}{2}\int e^u du = e^u = \boxed{e^{4\sqrt{y}}+C}$

6. $\int_{\pi/2}^{\pi} e^{\sin(q)}\cdot\cos(q)\,dq \quad u=\sin(q) \quad \frac{du}{dq}=\cos(q) \quad dq=\frac{du}{\cos q}$

upper limit $=\sin(\pi)=0$

lower limit $=\sin(\frac{\pi}{2})=1$

$\int_1^0 e^u\cdot\cos(q)\cdot\frac{du}{\cancel{\cos(q)}} = \int_1^0 e^u\,du = e^u]_1^0 = e^{\sin(0)}-e^{\sin(1)} = \boxed{1-e^{\sin(1)}+C}$

9. a. There is no function and its derivative seen to do substitution.

b. $\sin^2 x+\cos^2 x=1 \quad \sin^3 x=\sin x\cdot\sin^2 x$

$\sin^2 x = 1-\cos^2 x \quad \sin^3 x=\sin x\,(1-\cos^2 x) \quad \boxed{\int\sin x\,(1-\cos^2 x)\,dx}$

c. $u=\cos x \quad du=-\sin x\,dx$

☆ d. $\int\sin^3 x\,dx = \int\sin^2 x\sin x\,dx \quad u=\cos x \quad \frac{du}{dx}=-\sin x \quad dx=\frac{du}{-\sin x}$

$\int\sin^2 x\,\cancel{\sin x}\cdot\frac{du}{-\cancel{\sin x}} = -\int\sin^2 x\,du = \frac{1}{3}\cdot\cos^3(x)$?

e. $\sin(x)-\frac{1}{3}\sin^3(x)+C$