

Avoiding

the

Ineffective

Keyword

Strategy

sum

difference

remain



Try these meaningful alternative approaches to helping students make sense of word problems.

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Word problems are an essential component of providing a context for computation but are also recognized as an enduring struggle for students (Loveless et al. 2008; OECD 2015). Students are challenged to solve these story-based problems, often resorting to trial and error or to the practice of selecting the operation that represents “what we have been doing all week.” Even when students are given a problem that does not make sense—such as the infamous “If there are 25 sheep and 5 dogs in a flock, how old is the shepherd?” problem—students calculate a solution using nothing more than the numbers in the problem coupled with the knowledge of the operation used in the most recent lessons (Caldwell, Kobett, and Karp 2014). In a wonderful video by Robert Kaplinsky (2013) on how his eighth-grade students tried to solve a version of this problem, one student even articulates that she decided to divide on the basis of the lack of such words as *sum*, *product*, or *difference* in the problem.

An emphasis on seeking out *keywords* is frequently taught to students as an important or reliable approach to solving word problems. But this approach is a mistake. Let's be clear: We are not talking about a strategy of looking for essential information to make sense of the situation but instead a strategy where students seek *keywords* with a direct connection to a particular operation. For example, some might say (or some commercially available posters might encourage) that when you see the words or phrases *all together*, *in all*, or *more*, you should add; and the words *left*, *remain*, or *fewer* indicate the need to subtract. The problem is that these are not unfailingly true rules (see **fig. 1**).

When students were asked to explain their thinking, one student said, "When you see 'in all,' it makes it an easier problem 'cause you know it has to be addition. It can't be anything else." Consideration of the context and the relationship among the quantities expressed in the problem was ignored and thus caused a misinterpretation.

Another student, in describing the solution process, said, "First you find all the numbers in the problem. Then you look for an important word, like 'in all.' Then you use the chart to know what to do. Like, 'in all' just tells you that it is the whole bunch of stuff. Put together. That's why it says, 'in all!'"

Some *keywords* are clearly mathematical in nature, such as *product* and *sum*, but others (e.g., those just mentioned as well as *of*, *left*, and *altogether*) are not specific mathematical terminology. Students are taught that if they see these words to respond with a particular operation, yet that message cannot be generalized and is confusing. For example, research by Prediger (2011) found that when students were questioned about their approach to selecting an operation for a fraction word problem and given an opportunity to select multiple words indicating the operations, 56 percent revealed that when they see the word *of* in the problem, that is a signal to multiply. Interestingly, 51 percent suggested the word *of* also means to divide, and 33 percent indicated it can mean subtraction. Schoenfeld (1982) found that students exhibited a wrong-operation error when they subtracted when a word problem included a scenario about Mr. Left!

To some degree, the initial use of a keywords strategy is reinforced by basic story problems that are essentially computation wrapped in words. These formulaic and straightforward problems turn up in textbooks and consequentially appear to be successfully solved with keywords (Sulentic-Dowell, Beal, and Capraro 2006). This issue is further compounded because in primary grades students are often working within only simple, one-step word problems, which are much easier to form in ways in which keywords strategies can appear successful. For many years, researchers and mathematics educators have alerted practitioners in elementary schools and teacher educators in mathematics methods courses to avoid using the keywords strategy (e.g., Clement and Bernhard 2005; Hegarty, Mayer, and Monk 1995; Heng and Sudarshan 2013; Karp, Bush, and Dougherty 2014; Sowder 1988). Here are five limitations that explain the reasoning behind avoiding this strategy (adapted from Van de Walle, Karp, and Bay-Williams 2019):

FIGURE 1

Teaching students to find keywords as a problem-solving strategy is a mistake because these are not unfailingly true rules, as this student work demonstrates.

(a)

Marik has 3 packages of crayons. There are 11 crayons in each package. How many crayons does he have in all?

he has 14 $11 + 3 = 14$

(b)

Marik has 3 packages of crayons. There are 11 crayons in each package. How many crayons does he have in all?

$\begin{array}{r} 11 \\ + 3 \\ \hline 14 \end{array}$ He has 14 crayons in each package.

(c)

Ms. Adan bought 3 boxes of crayons with 8 crayons in each box. Mr. Logan bought 4 boxes of crayons with 6 crayons in each box. Who has more crayons? How many more crayons?

$\begin{array}{r} 18 \\ + 3 \\ \hline 21 \end{array}$ Ms. Adan has more crayons than Mr. Logan. She has 21 and he has 24. $\begin{array}{r} 16 \\ + 4 \\ \hline 20 \end{array}$

1. A keywords strategy does not necessitate any attempt to make sense of the actual problem presented, which negates the key feature of understanding the problem as advocated in the first of the Standards for Mathematical Practice (SMP 1, CCSSI 2010). We know that mathematics is unlike general reading, in which one or more words can be skipped and the “gist,” or essence, of the paragraph can still be distilled. Instead, mathematics requires the analysis of each word and the ability to identify the structure of the problem. Such questions as the following are considered: Is this word problem a result-unknown problem? Am I trying to find an unknown group size? The goal of a word problem is to have students practice connecting contextual situations of word problems to common additive (addition and subtraction) and multiplicative (multiplication and division) structures.

2. Students are incorrectly influenced by keywords that are taken out of the full context. For example, in a problem such as the following, students often will suggest a total of nineteen books because the prob-

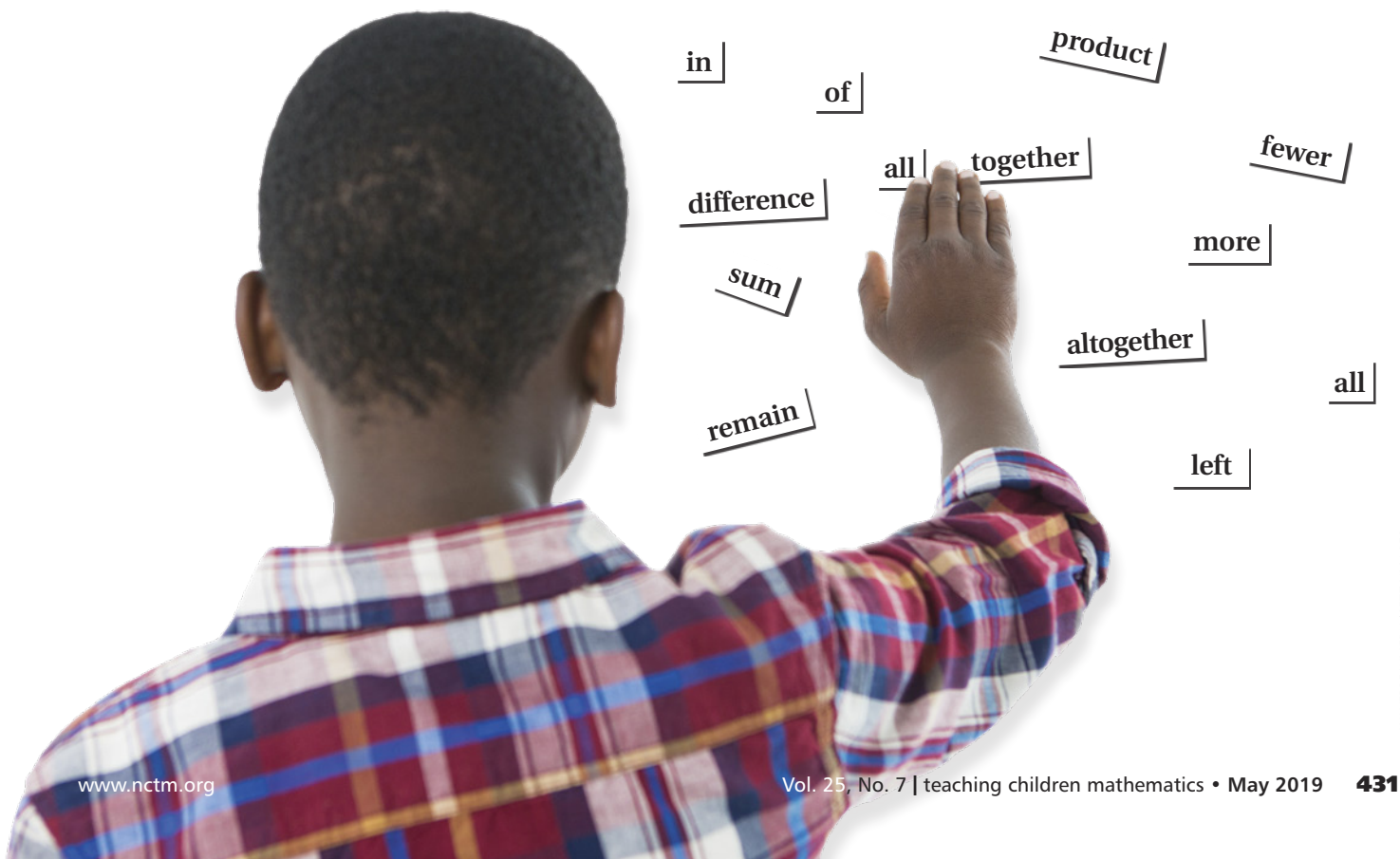
lem states *in all*, which gives students an immediate clue to add the numbers (Drake and Barlow 2007/2008):

There are seven cartons of books in the classroom. Each carton holds twelve books. How many books are there in all?

3. Many problems do not include keywords, and when students are faced with a problem without keywords, they are lost. They have no effective strategy. Students cannot discern how to enter the problem. For example, consider the problem below:

Forty-five children ride to school. If eighteen take the bus, how many come by car?

4. Keywords cannot be used with multistep problems, which begin when two-step problems are introduced (in second grade in CCSSI 2010). When problems have two or more steps and students rely only on keywords, they cannot infer which operation to perform first from simply identifying keywords. This approach often results in students completing steps that do not make sense within the context.



5. Students using a keywords approach are not practicing how to read mathematical situations and make sense of them using prior knowledge. From being taught a keywords strategy, students glean the implications that “you don’t have to read; you don’t have to think. Just grab the numbers and compute” (Hyde 2006, p. 3).

What now? Moving away from a keywords strategy

The work of Jordan and Hanich (2000) revealed that even when problems were read to students to avoid reading challenges, students with weak reading skills still performed at a lower level than other students. Instead of looking for ways to bypass or work around these challenges, teachers must deploy strategies that support students’ sense making—including their heightened awareness and use of operation sense. We advocate for the use of strategies and modifications of problems in place of the ineffective and harmful keywords strategies so that students become more familiar with types of problems and ways to think about problem situations.

- **Increase readability:** Researchers have identified that the readability of word problems has an effect on students’ performance, particularly for low-SES and low-proficiency students (Walkington, Clinton, and Shivraj 2017). This quality of *ease of reading* can be influenced by the length of the problem, the difficulty of the words used, and whether second-person pronouns that directly address the reader, such as *you* or *your*, are used. These researchers found that modifying problems by changing them to second-person pronouns, placing the reader into the problem, such as “round your answer” or “give your answer,” makes the situation more concrete and more easily accessed by learners.
- **Ensure topic relevance:** It is common sense, but if a problem is relevant to students, they will better understand the context, are more likely to be engaged, and have the potential to be more successful.
- **Support reading and understanding of mathematics words:** To develop the dis-

inction between words that have different meanings in mathematical situations than in conversation, students should learn these sometimes-confusing words. Vocabulary is at the core of content literacy. This emphasis on interpreting words that are spelled the same but have different meanings in mathematics includes such words as *table, face, product, mean, degree, similar, digit, even, odd, series, right, yard, volume, factor, base, foot, expression*, and *hand* (of a clock). Prepare students for these words in advance so that the focus of instruction can move toward understanding the situation in the word problem rather than understanding how language is used specific to mathematical situations.

- **Use concrete materials rather than abstract words:** Using manipulatives can help support students’ thinking as they use concrete materials to represent the situation in the context of the word problem.
- **Have students imagine the situation:** Instruct students to think about and articulate the situation in the problem. Get them to focus on the quantitative relationships rather than ignoring the context and focusing on a quick numerical answer.

Modeling may be viewed as the link between the “two faces” of mathematics, namely its grounding in aspects of reality, and the development of abstract formal structures. (Greer 1997, p. 300)

- **Employ schema-based instruction:** Having students carry out the action in the problem with a schema supports the identification of the problem structure and thereby the approach that leads to a solution (Jitendra and Star 2011). Students need to discuss the relationships inherent in these structures and need to describe the situation. In this way, they can meaningfully retrieve which operation to use by connecting the new problem to related schemas.
- **Have students put the problem into their own words:** Ask students to paraphrase the word problem to support their full comprehension of the situation and what is being asked.

- **Have students work backward:** Have students create their own word problems. For example, first give students an equation or picture, and then have them write a word problem to match. For students to fully understand the way actions in word problems are written, they need to become writers of the problems.
- **Act out the problem:** Especially for younger students, role playing the situation in the problems is a meaningful way to understand what is being asked. Although all students can benefit, this strategy can be particularly useful with students who are not yet efficient readers, writers, or drawers of mathematical situations.
- **Include an advanced organizer:** Moving the question to the beginning of the problem sends a message of what will be looked for. This approach particularly supports students who struggle (Thevenot et al. 2007). All the other information is then sorted according to this desired outcome. For example, the traditional form of word problems would be reworded to (or reread as): *How many more stickers does Emma have than Jack? Emma has 14 stickers. Jack has 8 stickers.* Sometimes this approach involves anticipation

FIGURE 2

A trajectory of sequenced actions, by grade bands, can frame students' growth and development in learning to make sense of problems.

K–5 trajectory of sequenced approaches to solving word problems

Kindergarten

1. Discuss what the problem is asking.
2. Act out the problem (teacher purposefully selects and uses problems that work well for role playing).
3. Teacher writes the corresponding equation until students can do so.
4. Student records the equation and the solution.
5. Determine the corresponding equation.
6. Record the equation and the solution, including the appropriate unit.
7. Explain your solution process orally and in writing, including why you chose it.

Grade 1

1. Discuss what the problem is asking.
2. Act out the problem in concrete ways, either role playing or using materials.
3. Represent the quantitative relationships in the problem by using graphic organizers or schema—such as Start, Change Result, Part-Part-Whole—or create a sketch or illustration to represent the problem.
4. Determine the corresponding equation.
5. Record the equation and the solution.
6. Record the solution, including the appropriate unit.
7. Explain your thinking orally or in writing, including why you chose the solution process and how you determined your answer is reasonable.

Grades 2–3

1. Identify important information.
2. Imagine the situation.
3. Act out the problem by role playing.
4. Strategically select a tool to model the problem, such as concrete materials, a schema, or the creation of a sketch or illustration as a representation of the problem.
5. Write the corresponding equation.
6. Record the solution, including the appropriate unit.
7. Explain your thinking orally or in writing, including why you chose the solution process and how you determined your answer is reasonable.
8. Check your computations. Is this the only answer, or are there other solutions? What is another solution strategy?

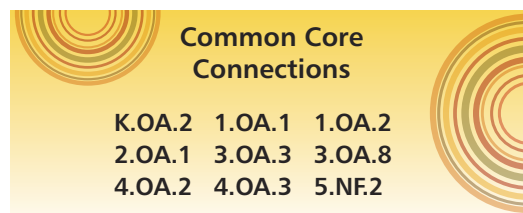
guides as a way to “scaffold text comprehension” (Adams, Pegg, and Case 2015, p. 501).

- **Give students a problem without a solution or without numbers:** Have students read a problem and then provide them with several solutions from anonymous students from another class. Sometimes pose problems that have no solutions (such as the classic Shepherd problem mentioned previously), or present solutions that are all incorrect. Also, pose word problems without numbers and then ask such questions as, “Should the answer be greater than the initial amount? Less than?” Engaging in this practice encourages students to focus on the context of the problem rather than on the numbers in the problem. Removing the numbers also creates parameters that help students determine if the answer they got is reasonable.
- **Give students a problem without a question:** Give students a word problem that presents only a situation. For example, *Jonas had some books. He gave Kaitlin seven books. He now has eight books.* Ask students to first identify possible questions that could be asked about the problem and then solve them.

Additionally, a purposeful, systematic approach should develop to meet SMP 1: *Make sense of problems.* We provide a trajectory of sequenced actions, by grade bands, to frame students’ growth and development (see **fig. 2**).

The goal is understanding

At the elementary school level, student success with word problems depends not on quickly computing a solution to the problem but instead on understanding the situation and identifying the right approach or correct operation. Working toward that goal through the use of multiple strategies will scaffold students’ attention to the structure of mathematics and how they can begin to see mathematics in the situations they encounter in life. We hope this article has convinced you to remove keywords strategies from your classroom instruction and instead focus on strategies to promote understanding and sense making, as described here.



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