MATH 212: Calculus II SP24 TQ LT14

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LT14: I can find the Taylor series of a function.
Both questions need to be completed to earn a check, but only one needs to be correct.

1. Find the first four terms of the Taylor series centered at $a = 0$ for the function $f(x) = \sin(2x)$ by using the definition or a known Taylor series, and write in expanded form.

definition $\sin(x) = \sum_{k=0}^{\infty} (-1)^k \frac{1}{(2k+1)!} x^{2k+1}$

therefore $\sin(2x) = \sum_{k=0}^{\infty} (-1)^k \frac{1}{(2k+1)!} 2x^{2k+1}$

$= 2x - \frac{1}{3!}2x^3 + \frac{1}{5!}2x^5 - \frac{1}{7!}2x^7$

2. The Taylor series for $\tan^{-1}(x)$ has summation formula

$$\sum_{n=0}^{\infty} \frac{x^{2n+1}}{2n+1}$$

Find the interval of convergence, where the Taylor series is equal to the function. ratios

$r_n(x) = \frac{x^{2n+1}}{2n+1} = \frac{x^{(2n+2)}}{2n+2} \times \frac{2n+1}{x^{2n+1}}$

$r(x)_{n+1} = \frac{x^{2n+1+1}}{2n+1+1} = \frac{x^{2n+2}}{2n+1+1} \cdot \frac{2n+1}{x^{2n+1}} = \frac{x^{2n+2}}{x^{2n+1}}$

$= x^{2n+2-(2n+1)}$

$= x^1$

then $\lim_{n \to \infty} \frac{x^{2n+2}}{x^{2n+1}} = \frac{\infty}{\infty} = 1$

hence. $-1 < x < 1$